Charmonium resonances from lattice QCD

based on work with the Hadron Spectrum Collaboration

David Wilson

hadspec
hadspec.org

DiRAC

DiRAC Day
12th December 2024

PRL Editors' suggestion: arXiv: 2309.14070 (7 pages)
PRD Editors' suggestion: arXiv: 2309.14071 (55 pages)

UNIVERSITY OF CAMBRIDGE

THE ROYAL SOCIETY

Strong QCD effects are everywhere - we need lattice QCD

- Electroweak physics - e.g. determining strong QCD parts needed for CKM elements: *B* decays, g_μ-2, $K \to \pi\pi$ (DiRAC - Edinburgh, Glasgow, Southampton, Cambridge, ...)

- Hadron Structure - deep inelastic scattering, parton physics (DiRAC - Edinburgh, ...)

- QCD at finite temperature (DiRAC - Swansea, ...)

- also useful for some strongly coupled beyond the Standard Model theories (DiRAC - Liverpool, Southampton, Swansea, Plymouth, ...)

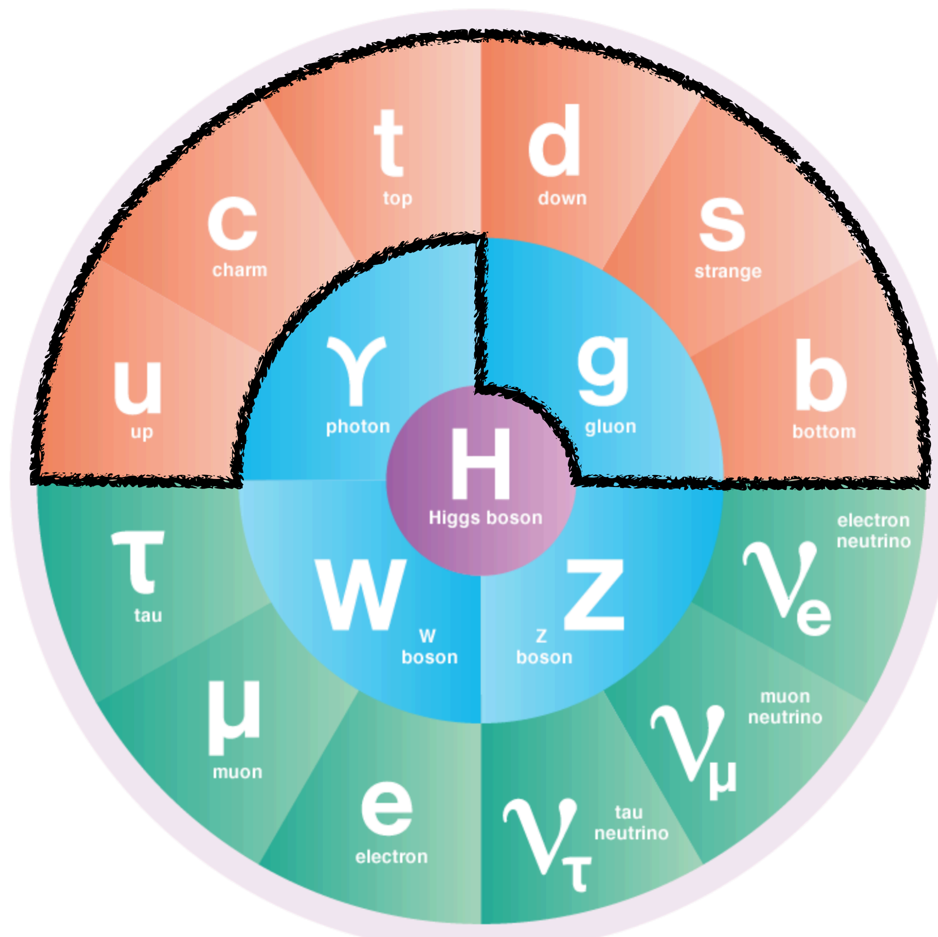
- **This talk:** Hadron Spectroscopy

  First principles QCD calculations of hadron decays and resonances (go beyond simple models)

  Big picture, general features (not yet ready for precision in most cases)

  Work concurrently with experiments: LHCb, GlueX, Belle-II, BES-III

  "Spectroscopy" shows up everywhere - eg the ρ in g_μ-2 calcs, $B \to K^* \mu\mu$, excited state effects in ground states, ...



[Fermilab/SLAC symmetrymagazine.org]

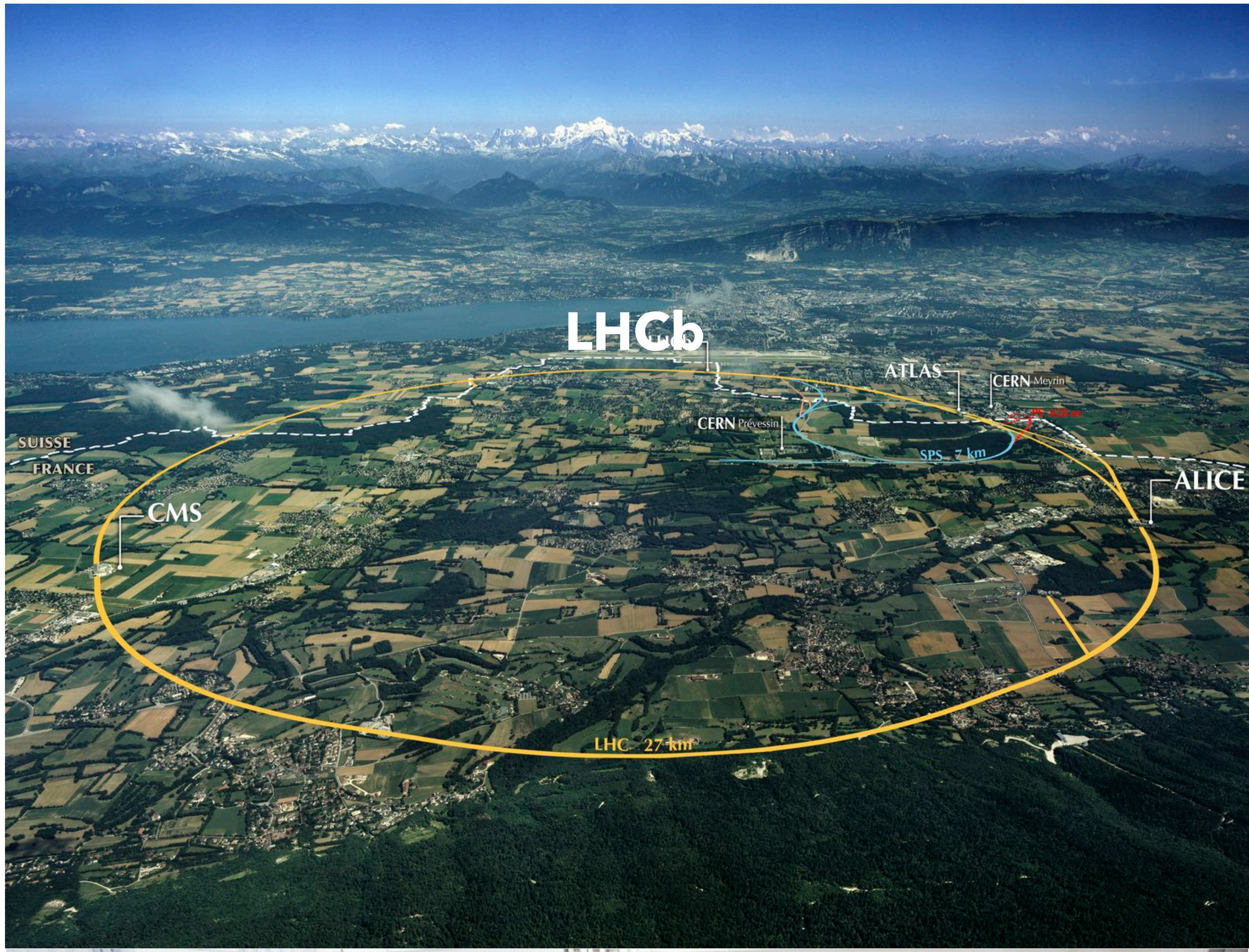
- Since 2003 there have been a wave of discoveries in hadron spectroscopy **Ds(2317), X(3872)**

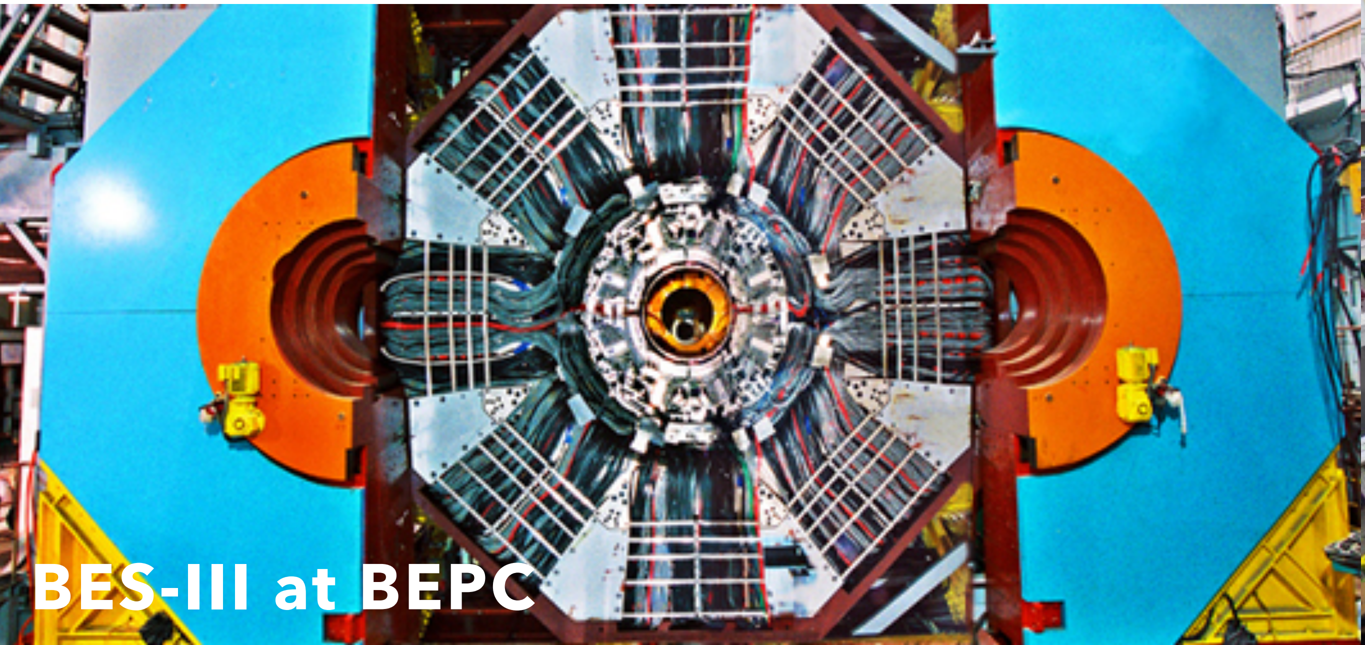
- Several active experiments

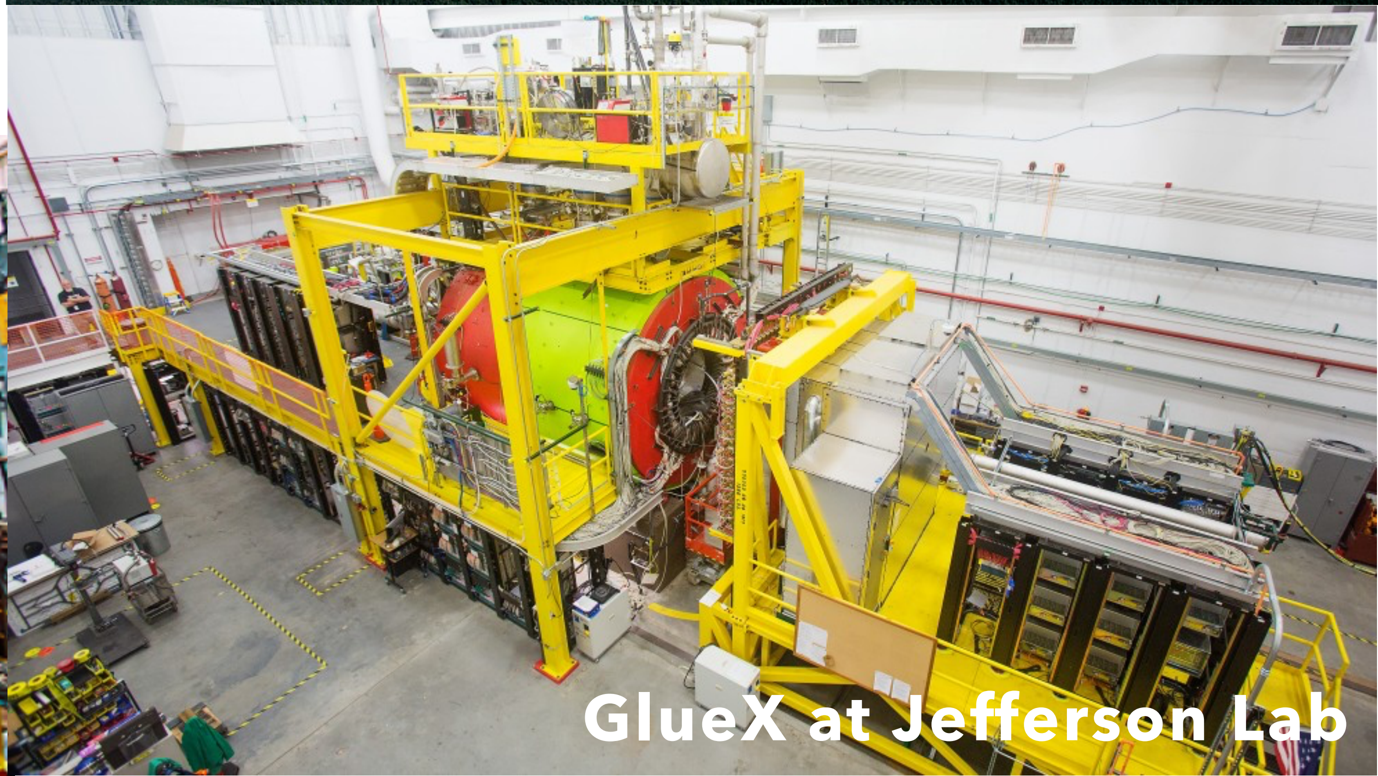
- We need to support the experimental efforts with theory predictions grounded in **QCD**

- Most new states involve the **charm** quark

+ Belle-II, Panda, EIC



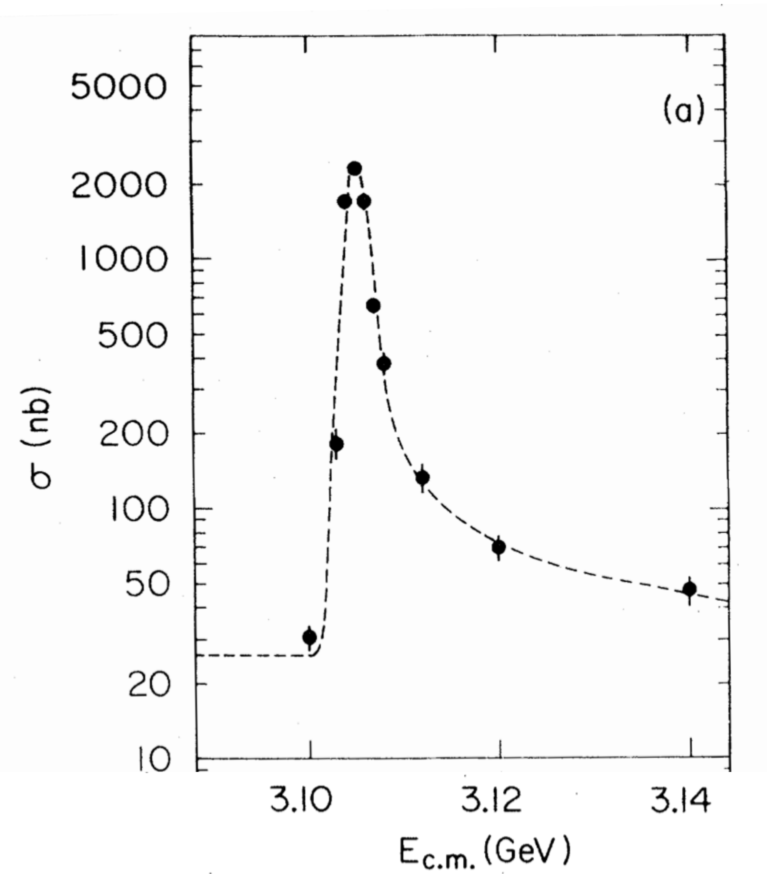




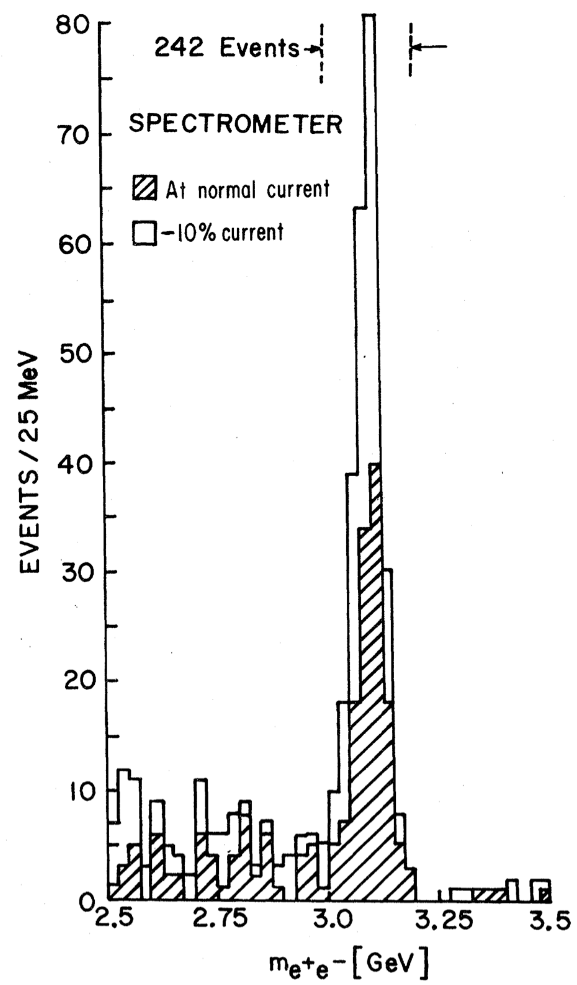
PRL 33 23,
2 Dec 1974

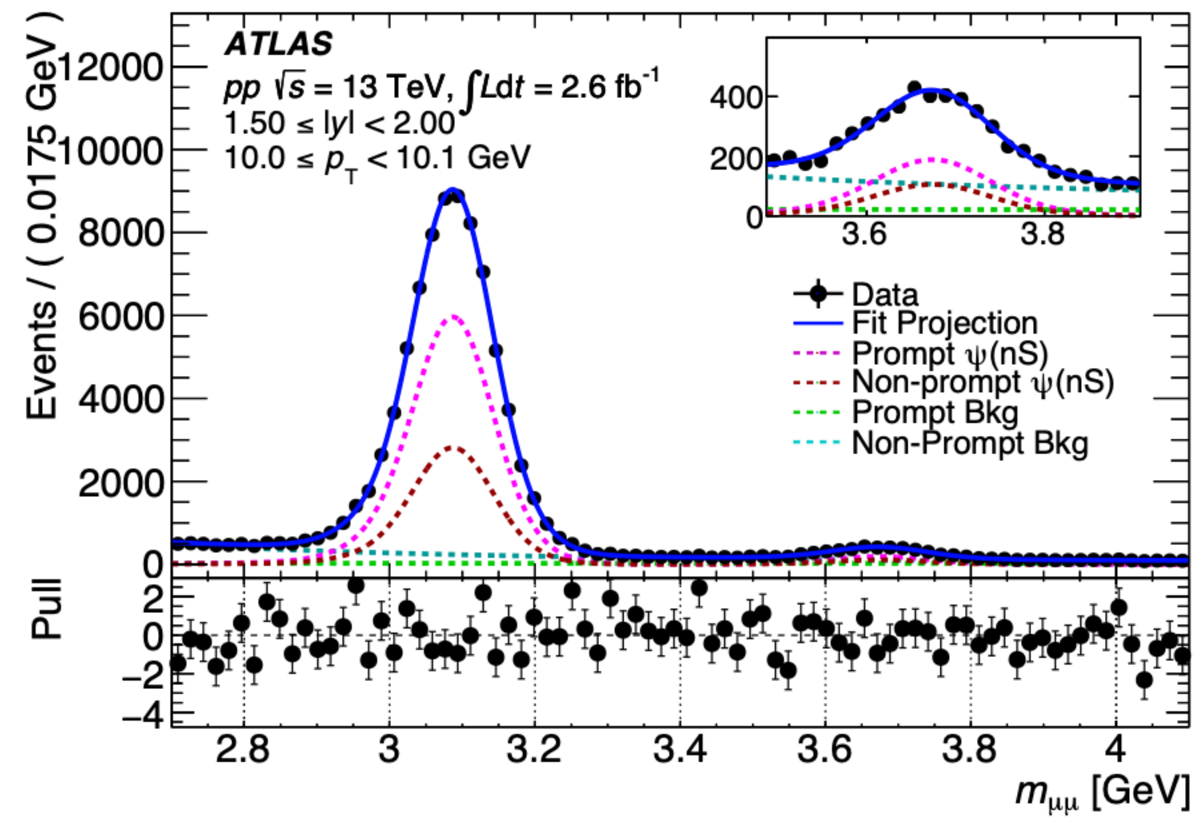
J/ψ

Augustin et al (SLAC)

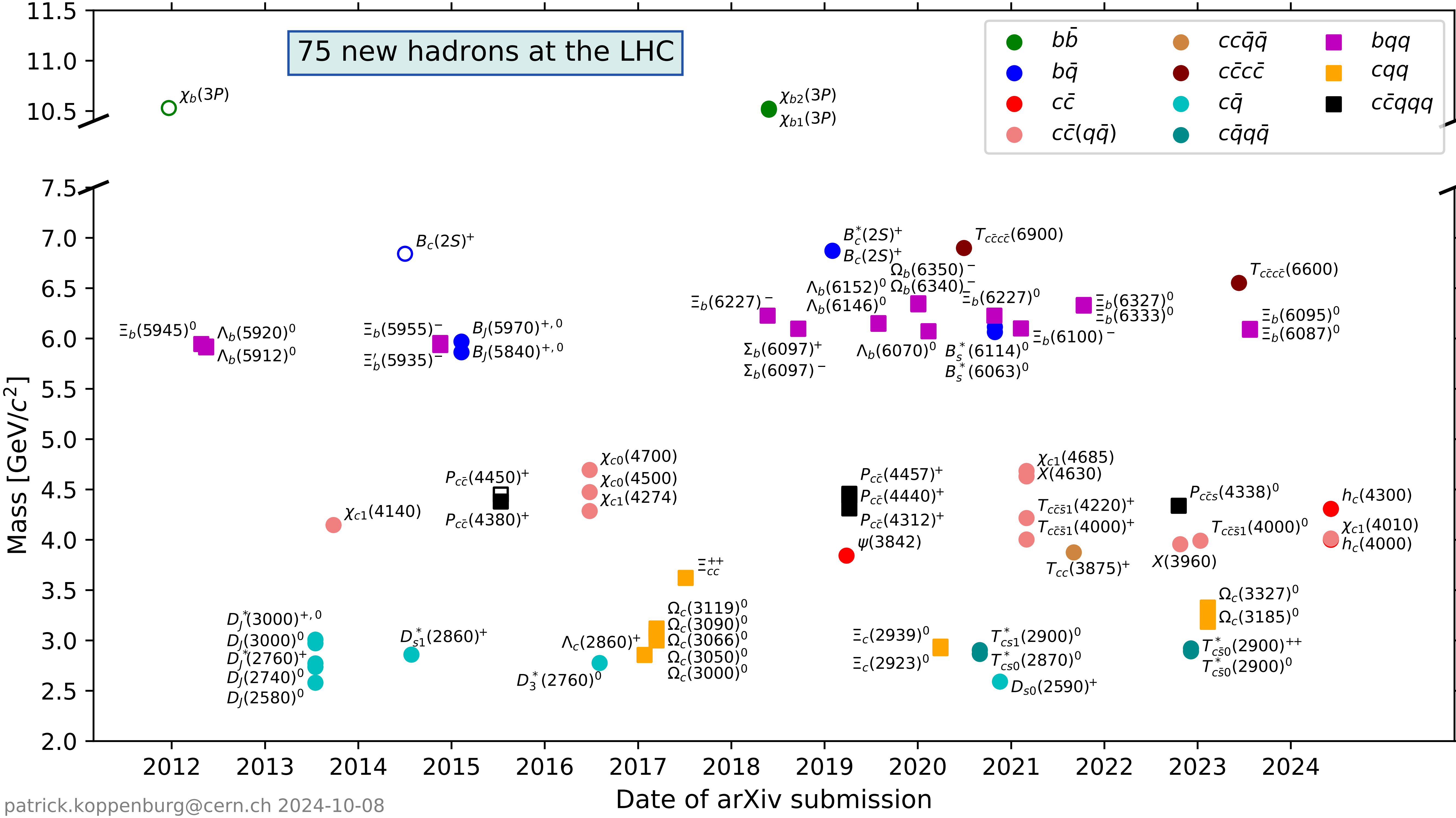


J. J. Aubert et al (BNL)





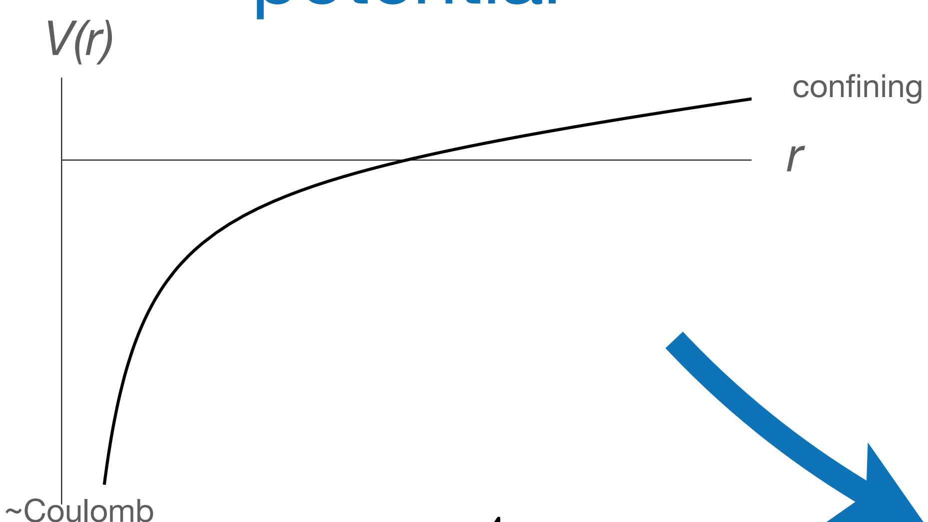
Several orders of magnitude more data


75 new hadrons at the LHC
Mass [GeV/c²]
Date of arXiv submission
bb̄
bq̄
cc̄
cc̄(qq̄)
ccq̄q̄
cc̄cc̄
cq̄
cq̄qq̄
bqq
cqq
cc̄qqq
χb(3P)
χb2(3P)
χb1(3P)
Bc(2S)+
Ξb(5945)0
Λb(5920)0
Λb(5912)0
Ξb(5955)−
Ξ′b(5935)−
BJ(5970)+,0
BJ(5840)+,0
Pcc̄(4450)+
Pcc̄(4380)+
χc1(4140)
χc0(4700)
χc0(4500)
χc1(4274)
Ξcc++
Ωc(3119)0
Ωc(3090)0
Ωc(3066)0
Ωc(3050)0
Ωc(3000)0
Λc(2860)+
D3*(2760)0
Ds1*(2860)+
DJ*(3000)+,0
DJ(3000)0
DJ*(2760)+
DJ(2740)0
DJ(2580)0
Ξb(6227)−
Σb(6097)+
Σb(6097)−
Λb(6152)0
Λb(6146)0
Bc*(2S)+
Bc(2S)+
Ωb(6350)−
Ωb(6340)−
Λb(6070)0
Tcc̄cc̄(6900)
Ξb(6227)0
Bs*(6114)0
Bs*(6063)0
Ξb(6100)−
Ξb(6327)0
Ξb(6333)0
Tcc̄cc̄(6600)
Ξb(6095)0
Ξb(6087)0
Pcc̄(4457)+
Pcc̄(4440)+
Pcc̄(4312)+
ψ(3842)
χc1(4685)
X(4630)
Tcc̄s̄1(4220)+
Tcc̄s̄1(4000)+
Tcc(3875)+
Pcc̄s(4338)0
Tcc̄s̄1(4000)0
X(3960)
hc(4300)
χc1(4010)
hc(4000)
Ξc(2939)0
Ξc(2923)0
Tcs1*(2900)0
Tcs0*(2870)0
Ds0(2590)+
Ωc(3327)0
Ωc(3185)0
Tcs̄0*(2900)++
Tcs̄0*(2900)0

charm quarks are heavy

➡ get an idea of the spectrum from the Schrödinger equation

potential

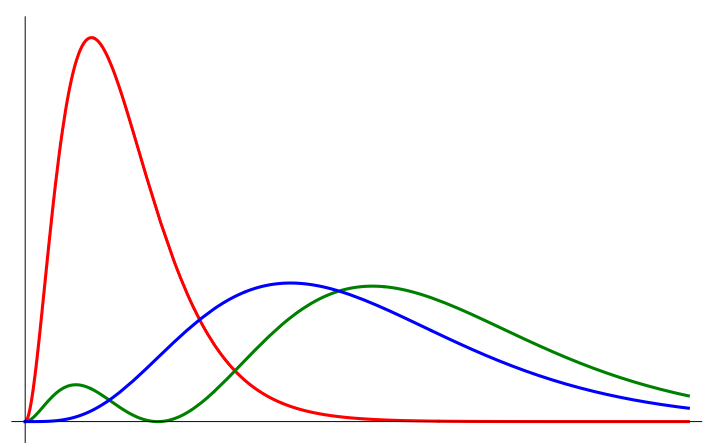


$$V(r) = -\frac{4}{3}\frac{\alpha}{r} + br$$

"QCD inspired"

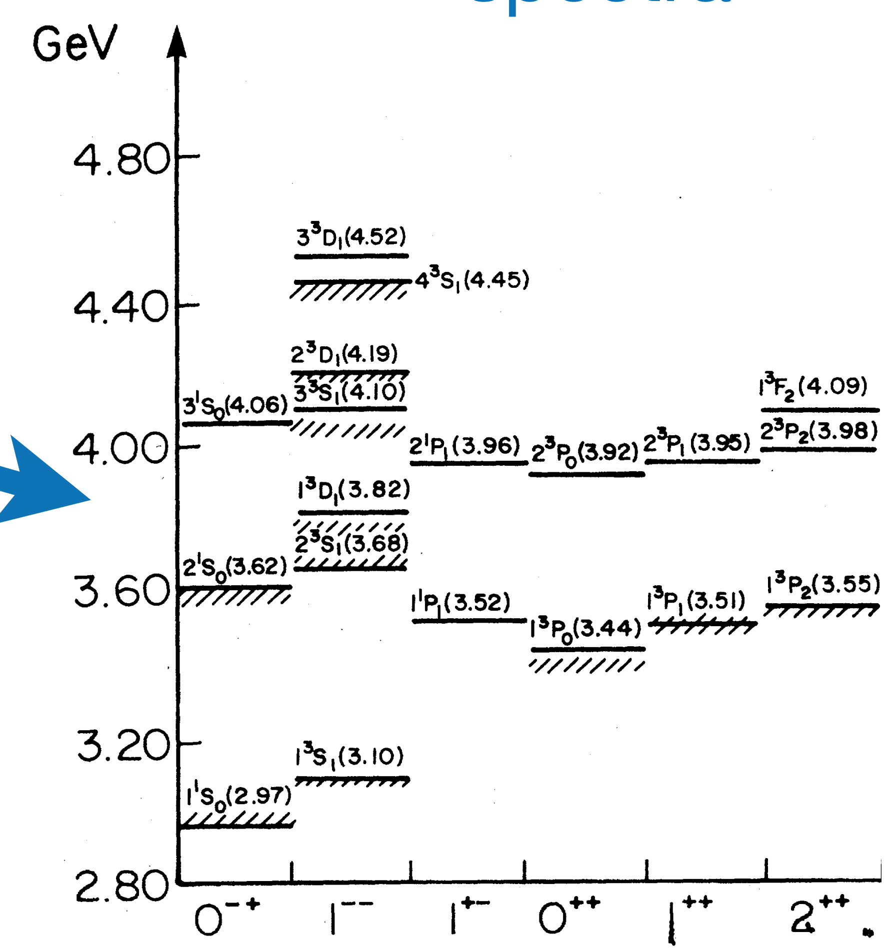
(but can be computed with static quarks on a lattice)

wavefunctions

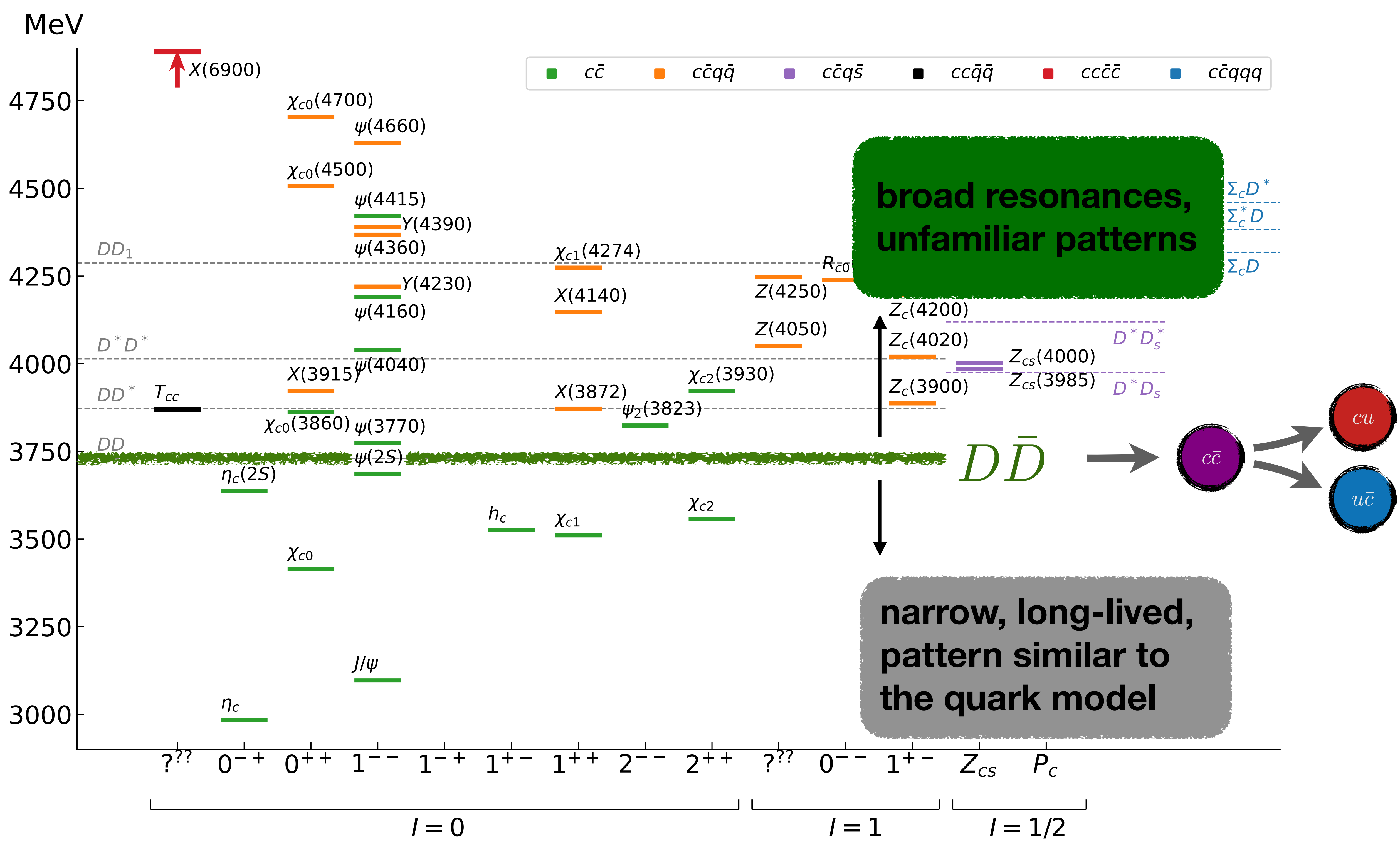


$$r^2 R^2(r)$$

spectra



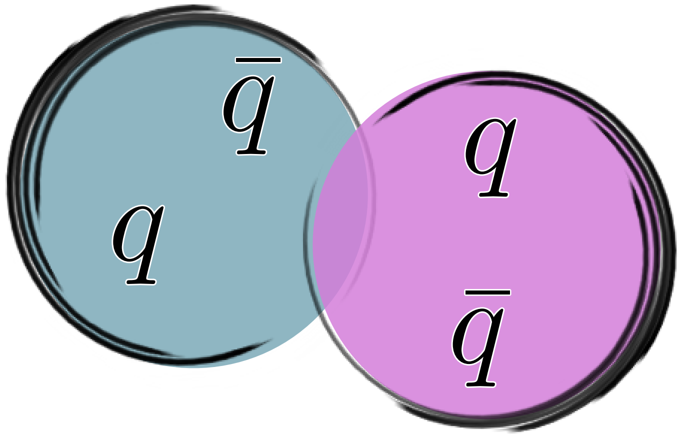
[Godfrey & Isgur 1985]



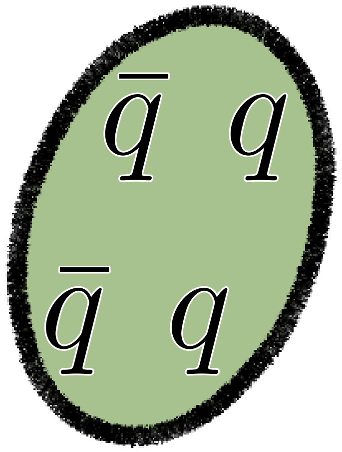
JPAC arXiv:2112.13436

beyond the simple quark model:

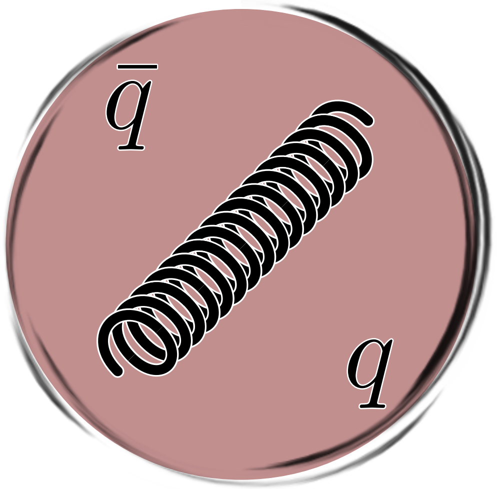
- molecules?



- tetraquarks?



- hybrids?



Numerically sample the path integral of QCD in a finite Euclidean volume

$$\langle \mathcal{O} \rangle \sim \int \mathcal{D}U e^{-S[U]}\, \mathcal{O}$$

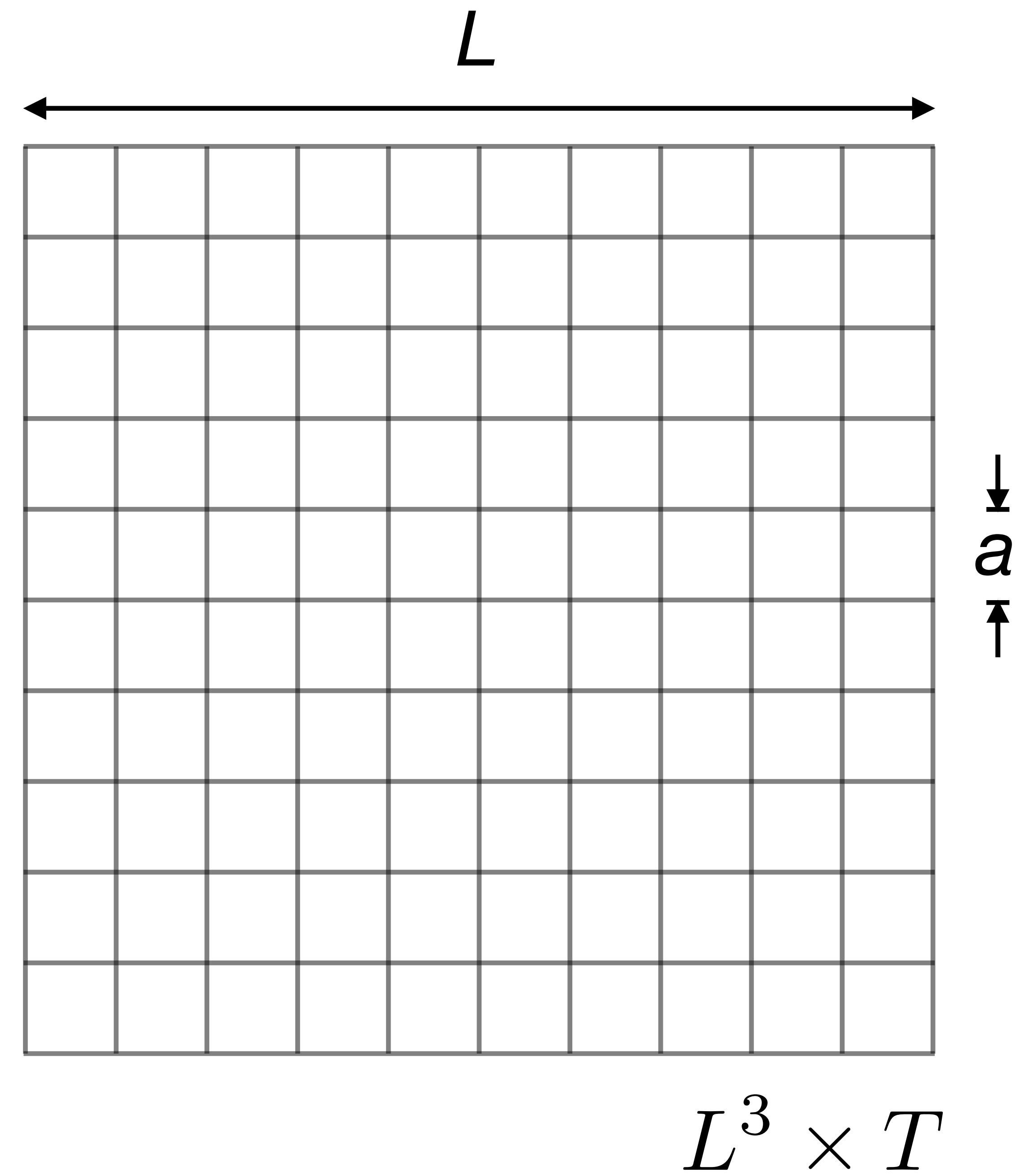
Compute euclidean-time finite-volume correlation functions

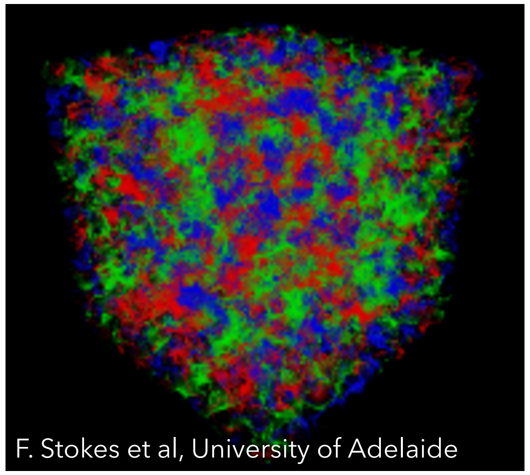
$$C(t) \sim e^{-E_{\mathfrak{n}} t}$$

No infinities due to:
a (lattice spacing, UV)
L^3 (lattice volume, IR)

Periodic BC: no continuum, discrete spectrum

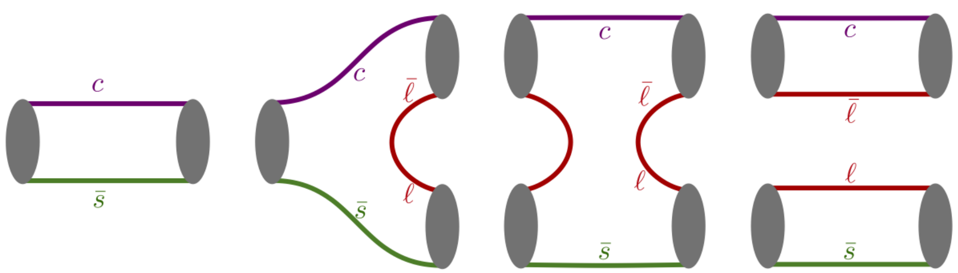
Usually m_π>140 MeV





Gauge ensemble generation.
MPI on (Intel) CPUs
(Most of gauge generation was run by collaborators on machines in the US)

Quark propagator inversions.
previously P100s (Cambridge),
A100s (Edinburgh and Cambridge)
In the Distillation method - we compute these once and reuse many times

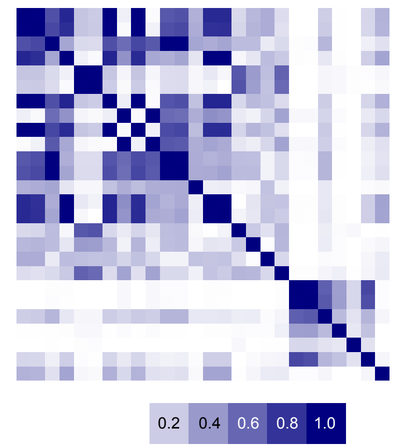


Wick contractions. Our current main use of DiRAC, Intel CPUs in Cambridge - Cascade Lake, Icelake, Sapphire Rapids

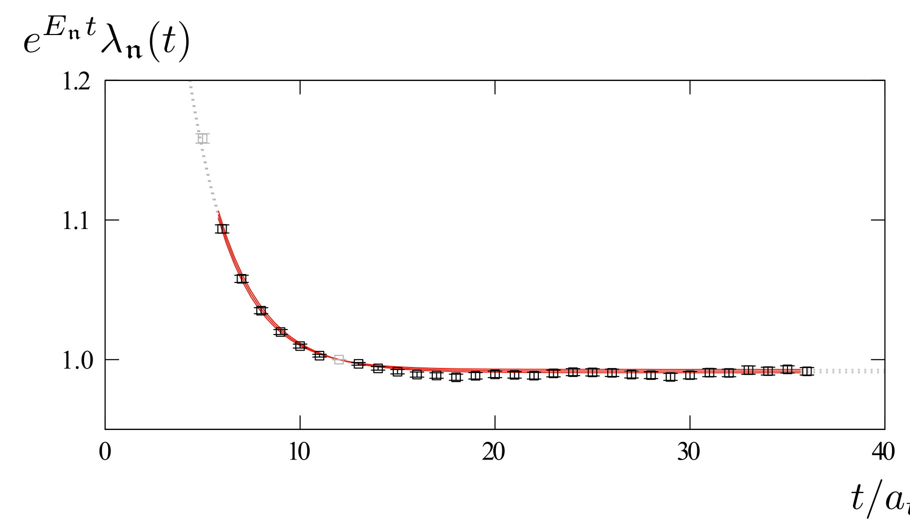
Correlation functions. Analysed on smaller machines.

Lattice QCD

Compute Correlation Matrix



Generalised Eigenvalue Problem



Operators

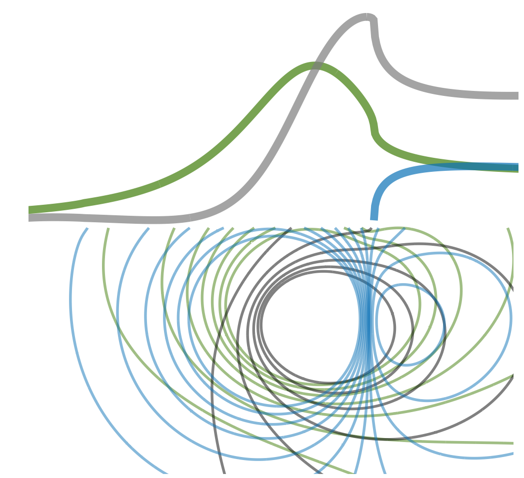
$$\mathcal{O}^\dagger \sim \bar{q}\Gamma q$$

$$(\pi\pi)^\dagger \sim \sum_{\vec{p}} (\text{CGs})\ \pi^\dagger \pi^\dagger$$

$$\pi^\dagger \sim \sum_i v_i^{(\pi)} \mathcal{O}_i^\dagger$$

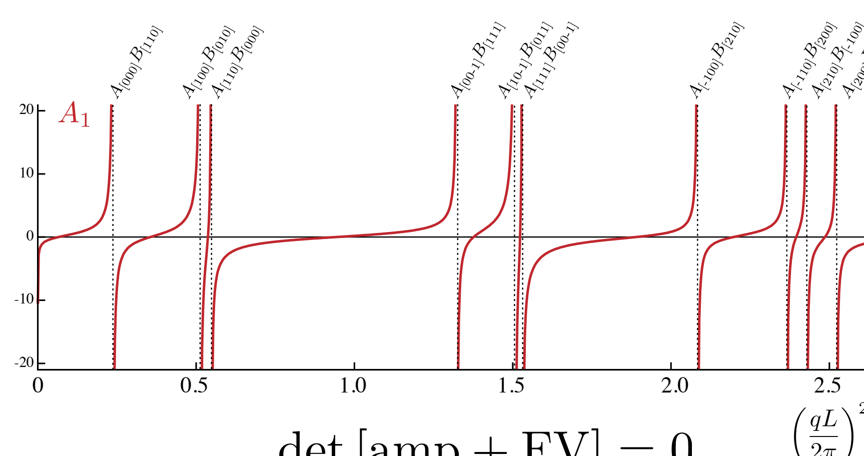
$$C_{ij}(t) v_j^\mathfrak{n} = \lambda_\mathfrak{n}(t) C_{ij}(t_0) v_j^\mathfrak{n}$$

Obtain Finite Volume Spectrum



Lüscher Quantisation Condition

Determine Scattering Amplitudes



Poles, Couplings

Review: Briceño, Dudek, Young,
https://doi.org/10.1103/RevModPhys.90.025001

$J^P = 0^+$

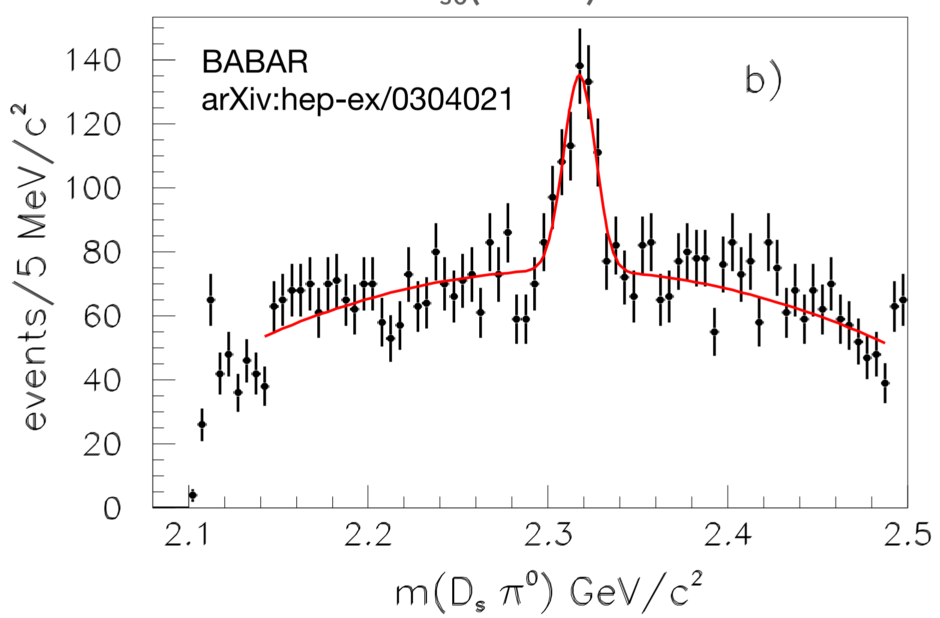
- $D_{s0}(2317)$ $c\bar{s}$
- $D_0^*(2400)$ $c\bar{l}$

compare : $J^P = 0^-$

- D_s m~1969 MeV
- D m~1870 MeV

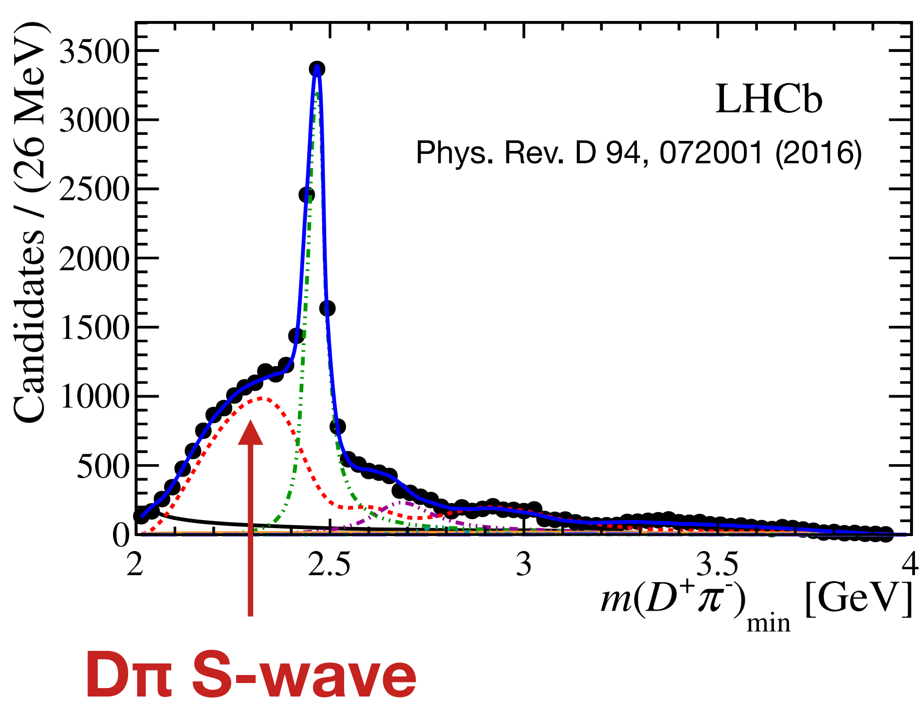
what is the mass ordering?
why are the masses so close?
why are the widths so different?

$D_{s0}(2317)$



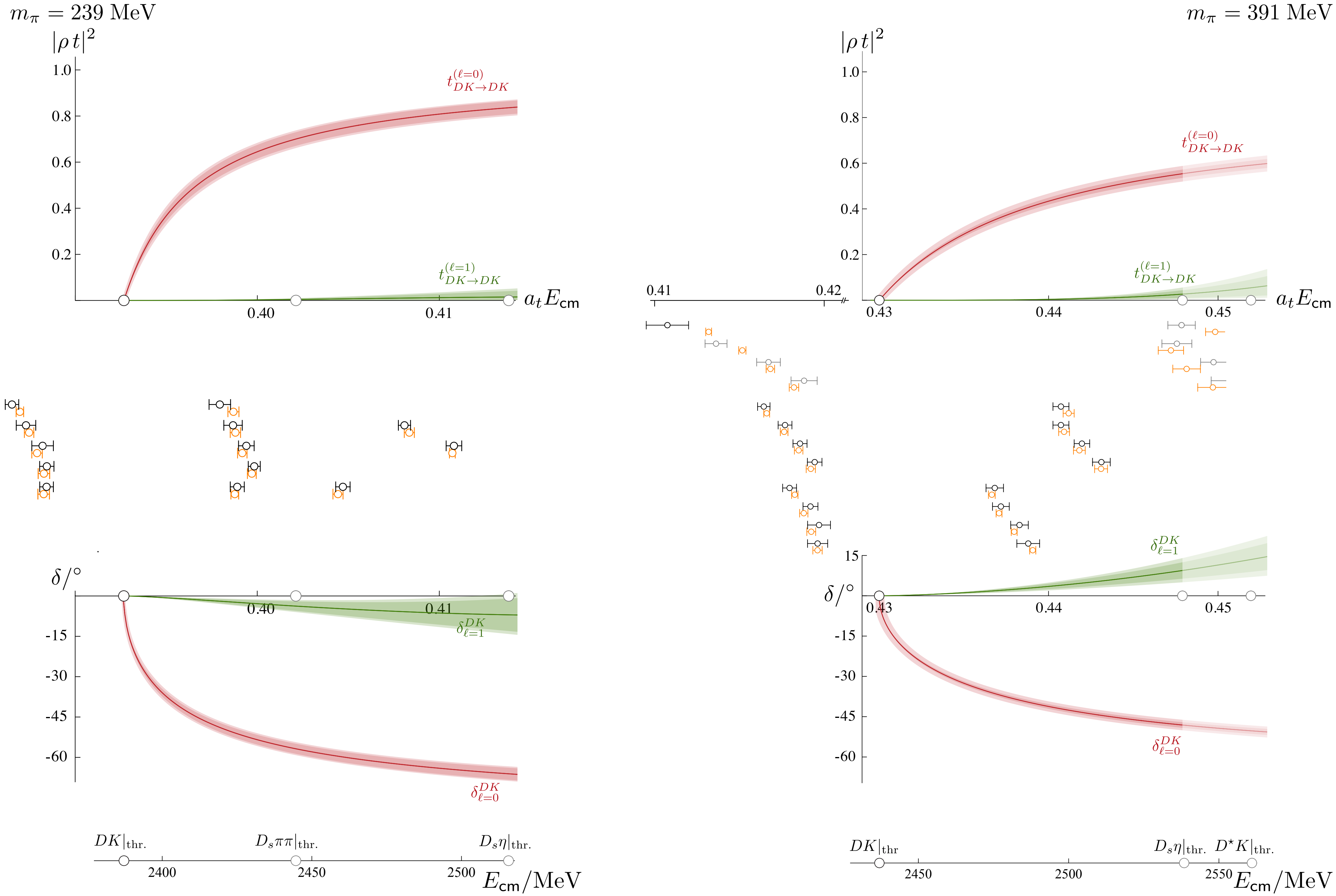
seen in an isospin breaking decay mode

$D_0^*(2400)$



G. Cheung et al (HadSpec), JHEP 02 (2021) 100 arXiv: 2008.06432

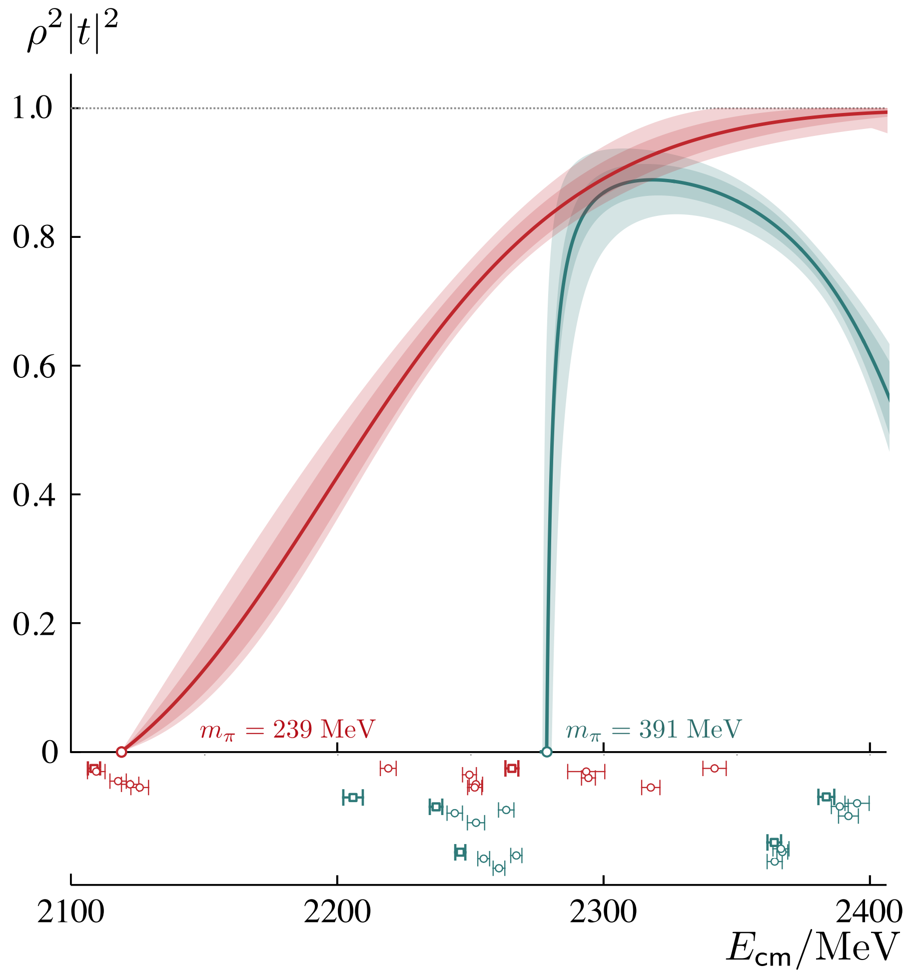
$D_{s0}(2317)$



bound states in DK amplitude at both masses

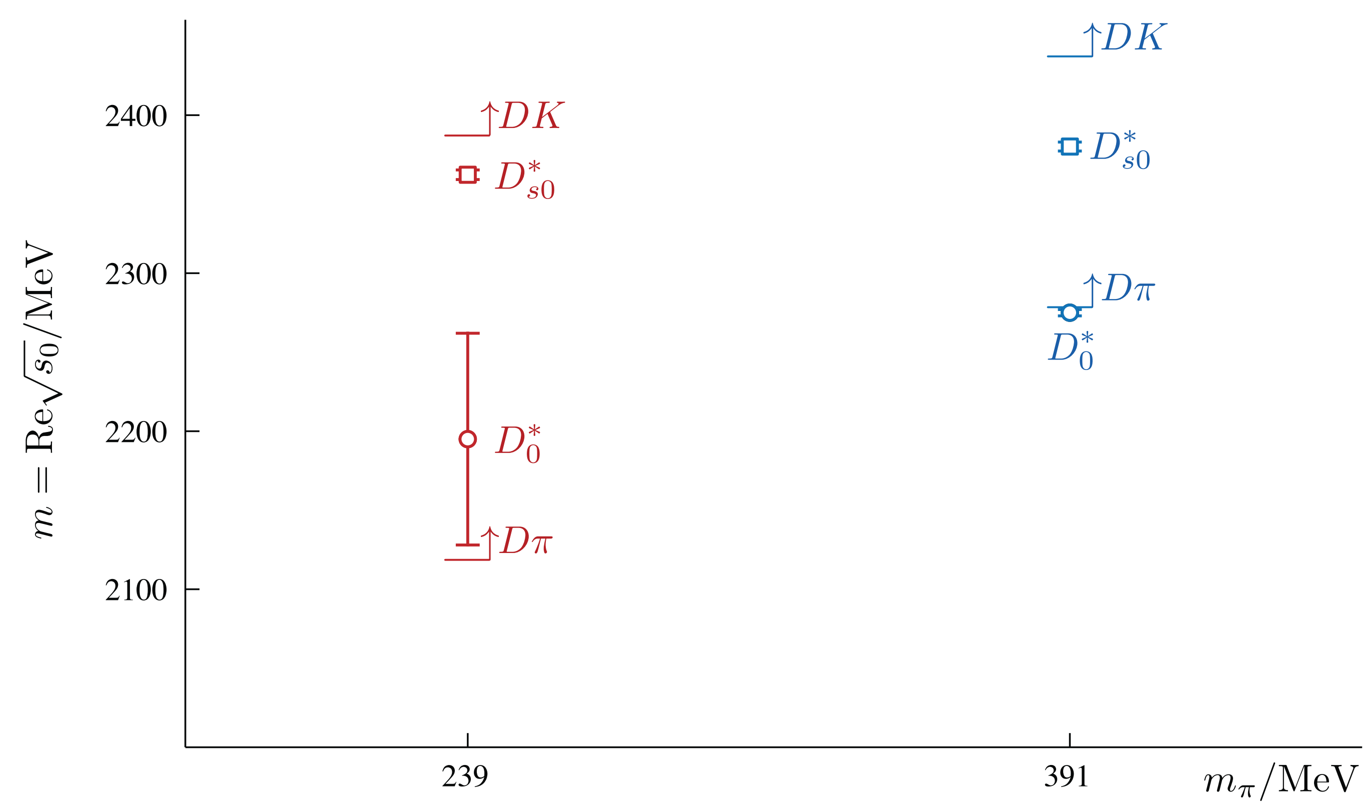
similar couplings c~1400 MeV

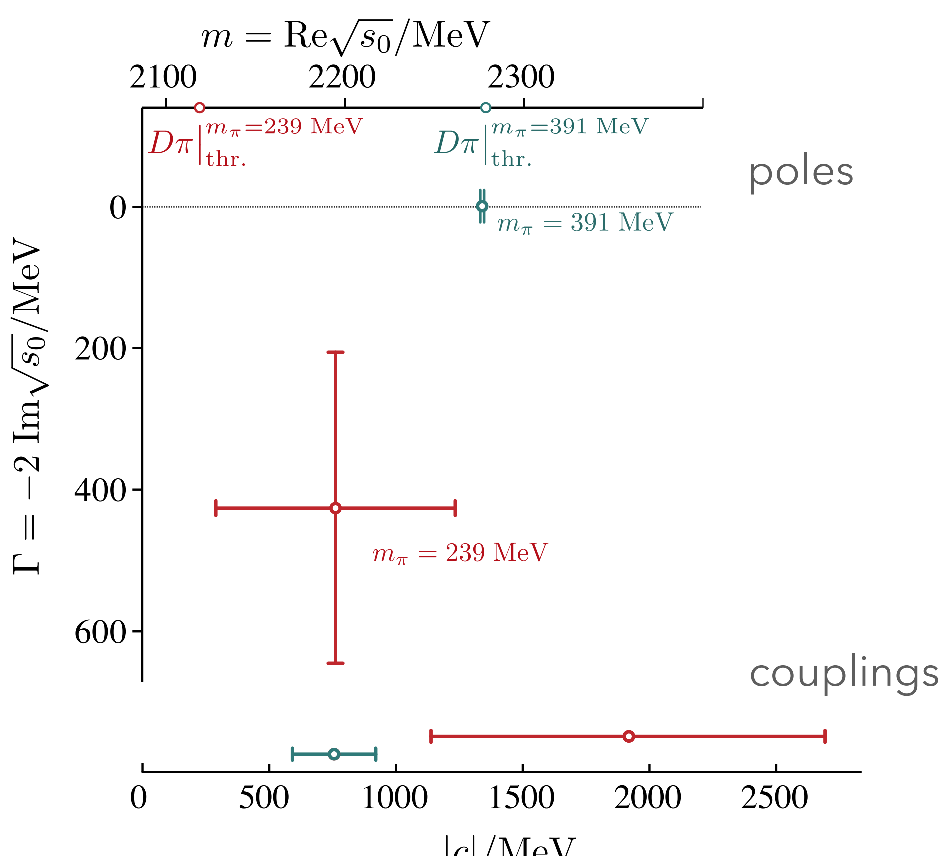
L. Gayer, N. Lang et al (HadSpec), arXiv:2102.04973



$$t \sim \frac{c^2}{s_{\rm pole} - s} \qquad \sqrt{s_{\rm pole}} = m \pm \tfrac{i}{2}\Gamma$$

suggestive of a much lighter D_0^* compared with the D_{s0}^*



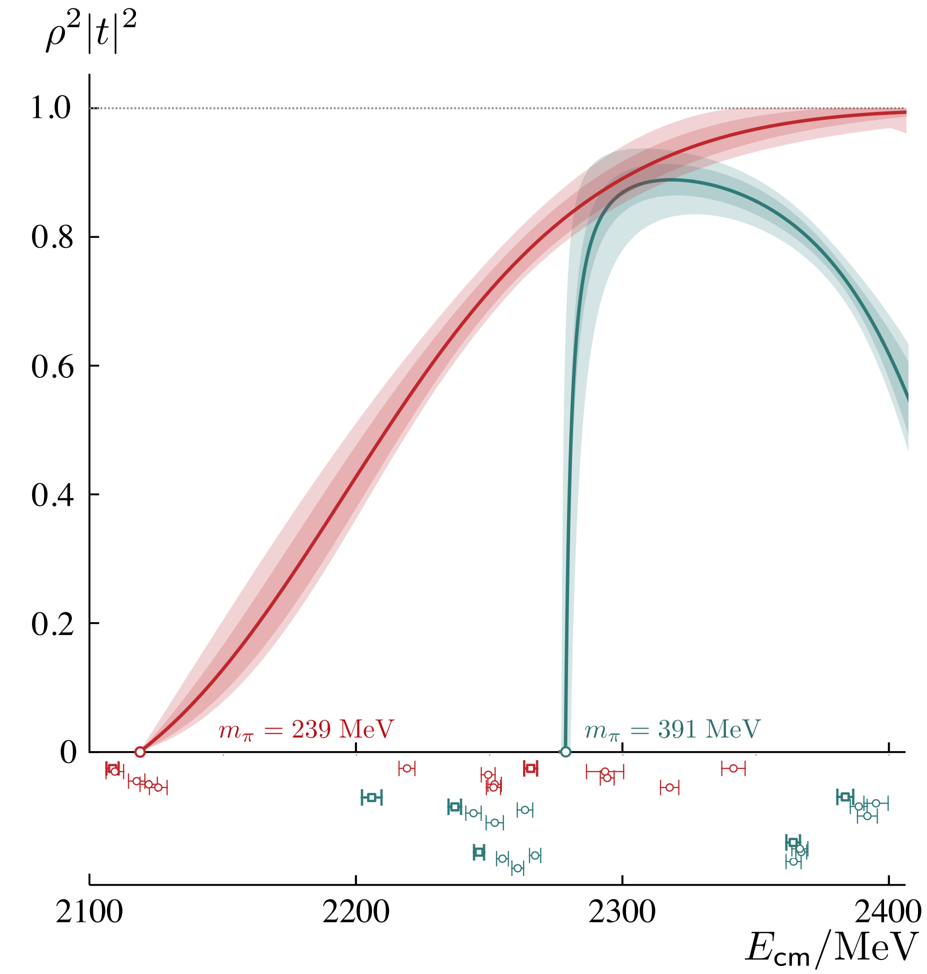


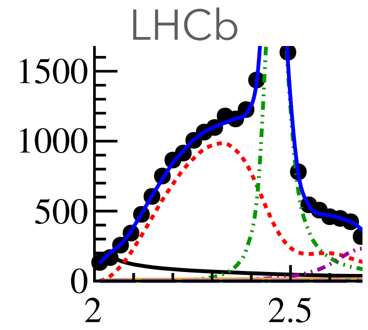
natural mass ordering: given light, strange constituents

likely hypothesis: D_0^* pole position is lower, m~2100-2200 MeV ?

see also LHCb data+ChiPT+unitarity: Du et al, PRL 126, 192001

L. Gayer, N. Lang et al (HadSpec), arXiv:2102.04973





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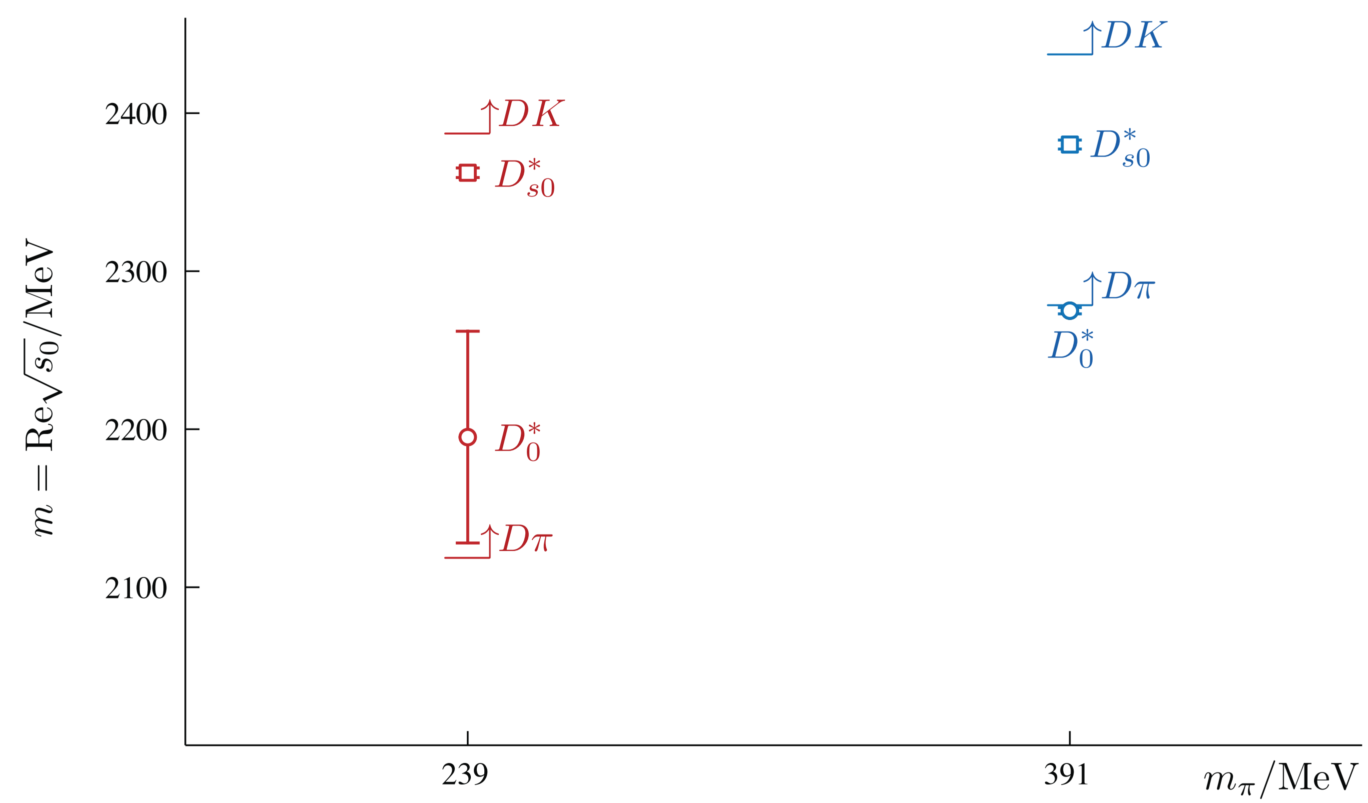

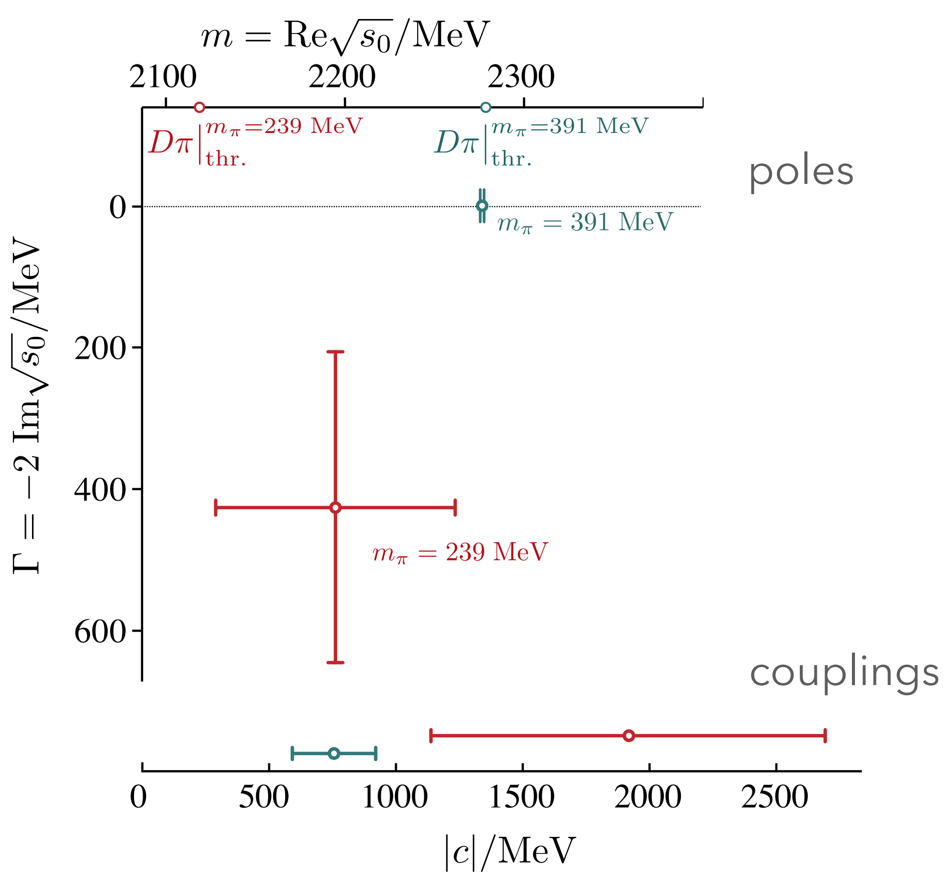

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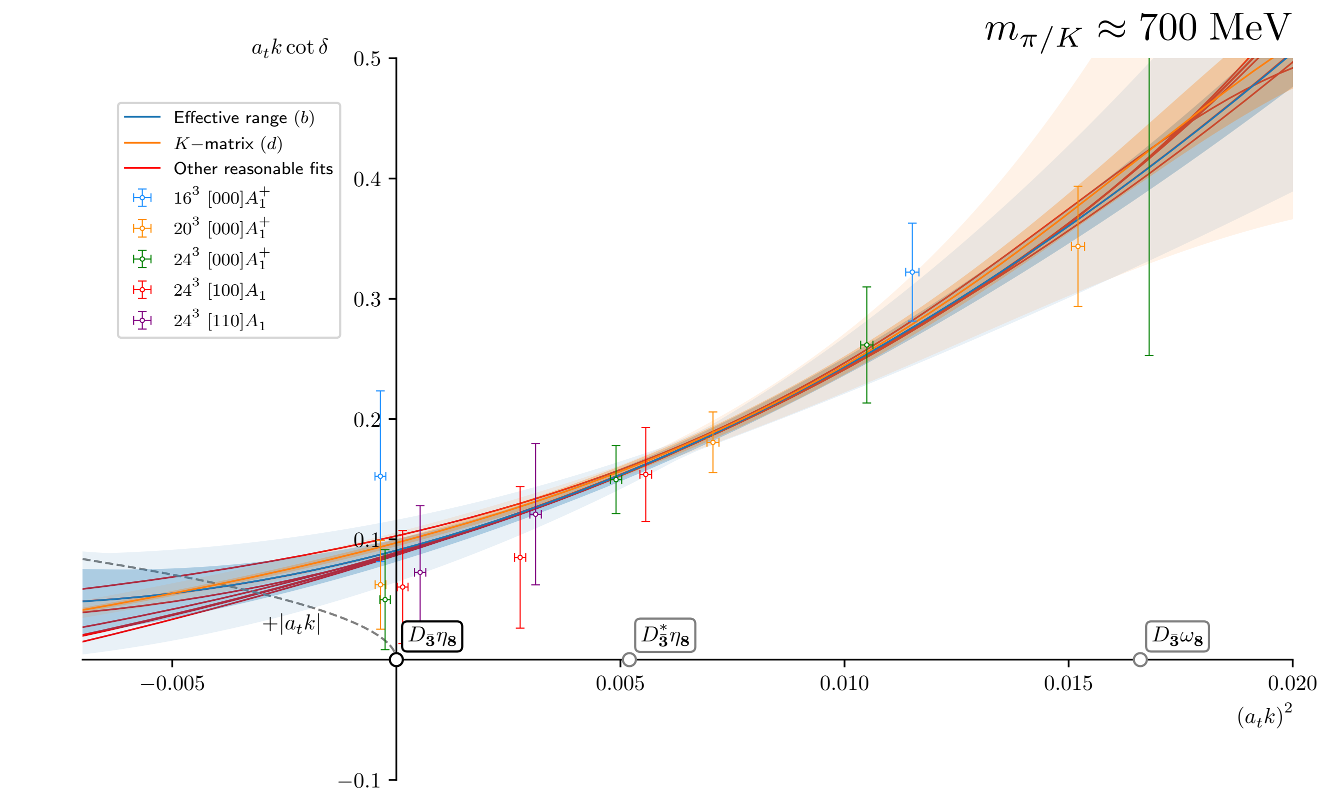
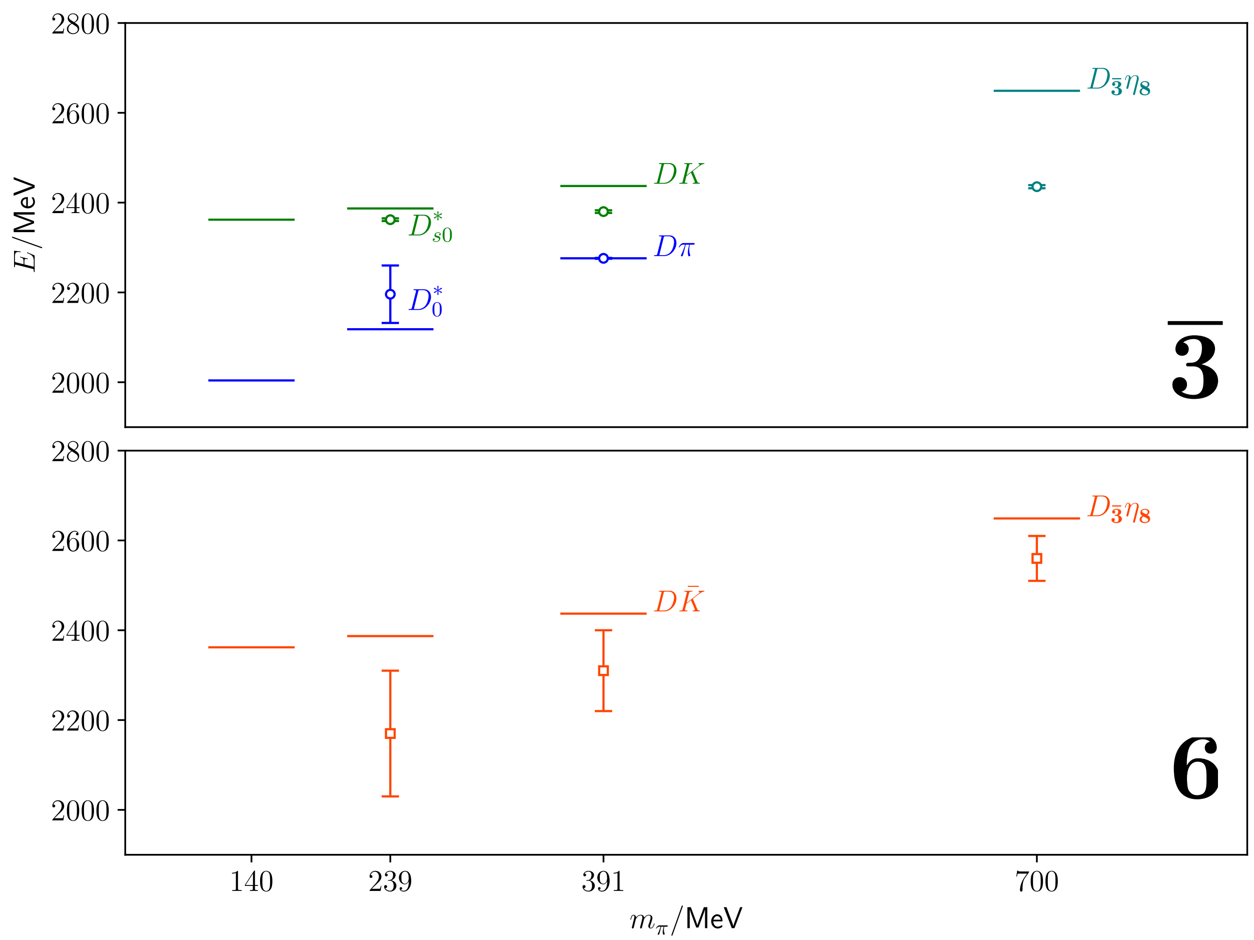
$m_u = m_d = m_s$





Dπ/DK scattering with SU(3) flavour symmetry
J. D. E. Yeo, C. E. Thomas, Wilson
arXiv:2403.10498

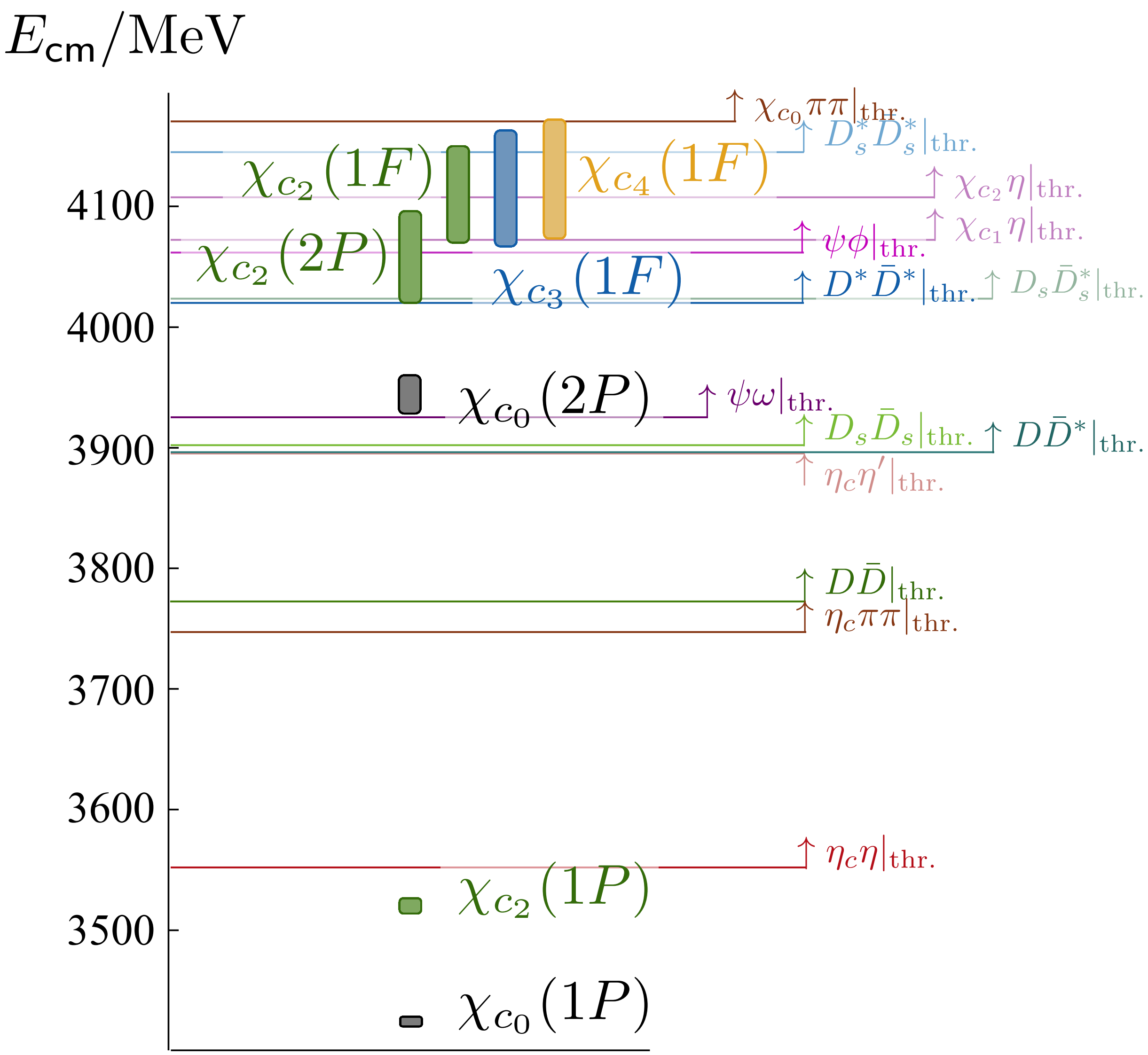
- S-wave interactions in flavour SU(3) 3bar, 6, 15bar
- Virtual bound state sextet pole
- Also deeply bound 3bar state, similar to Ds0(2317), much greater binding

SU(3) flavour:
D-meson and light meson

$$\bar{\mathbf{3}} \otimes \mathbf{8} \rightarrow \bar{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{15}}$$

$$\bar{\mathbf{3}} \otimes \mathbf{1} \rightarrow \bar{\mathbf{3}}$$

Previously:



spectra from qqbar operators only,
Liu et al JHEP 1207 (2012) 126

"HadSpec" lattices

anisotropic (3.5 finer spacing in time)
Wilson-Clover

L/a_s=16, 20, 24
m_π = 391 MeV

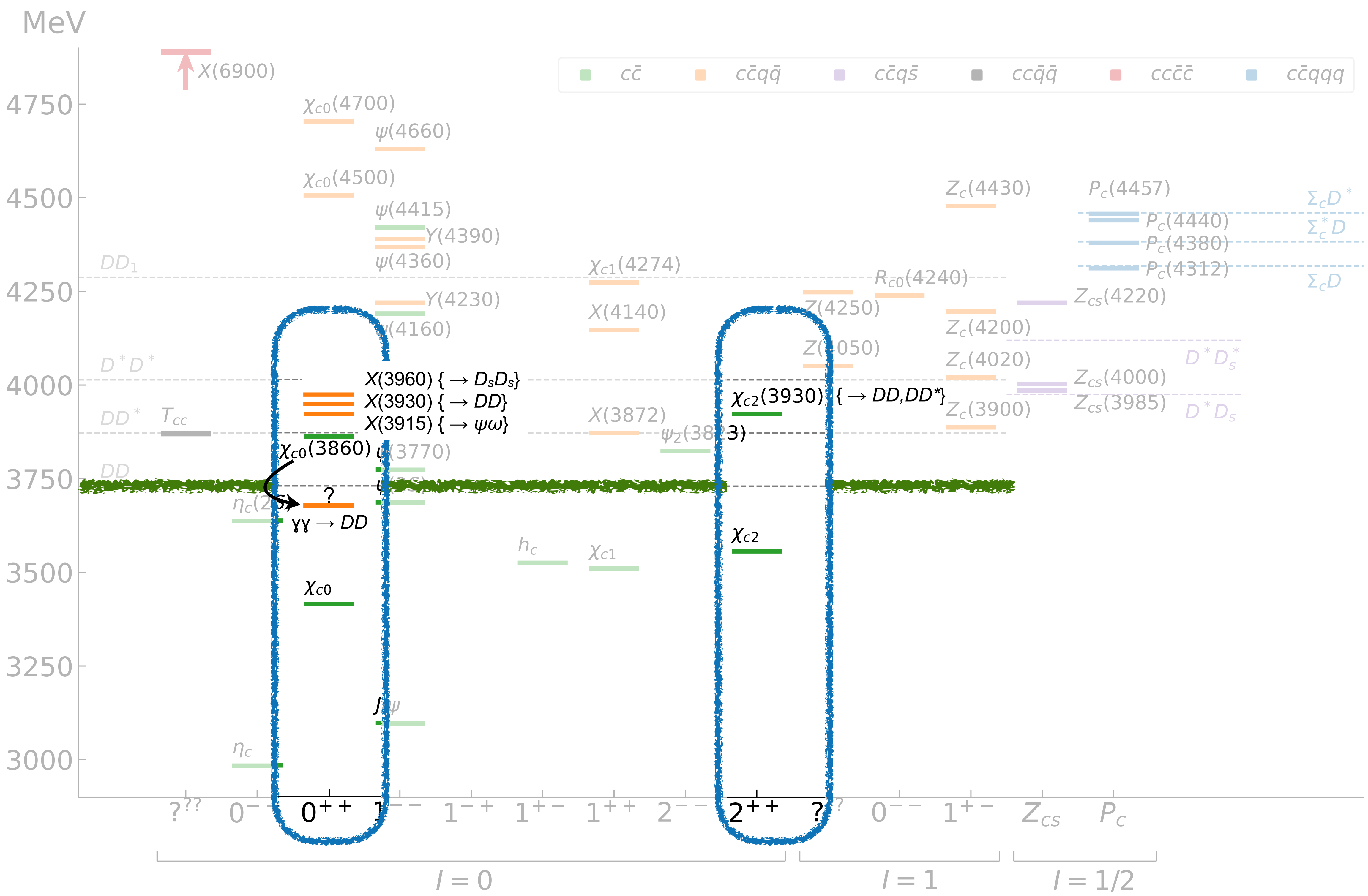
rest and moving frames

N_f = 2+1 flavours
all light+strange annihilations included
no charm annihilation

using *distillation* (Peardon *et al* 2009)
many channels, many wick contractions

This study: Meson-meson + qqbar ops

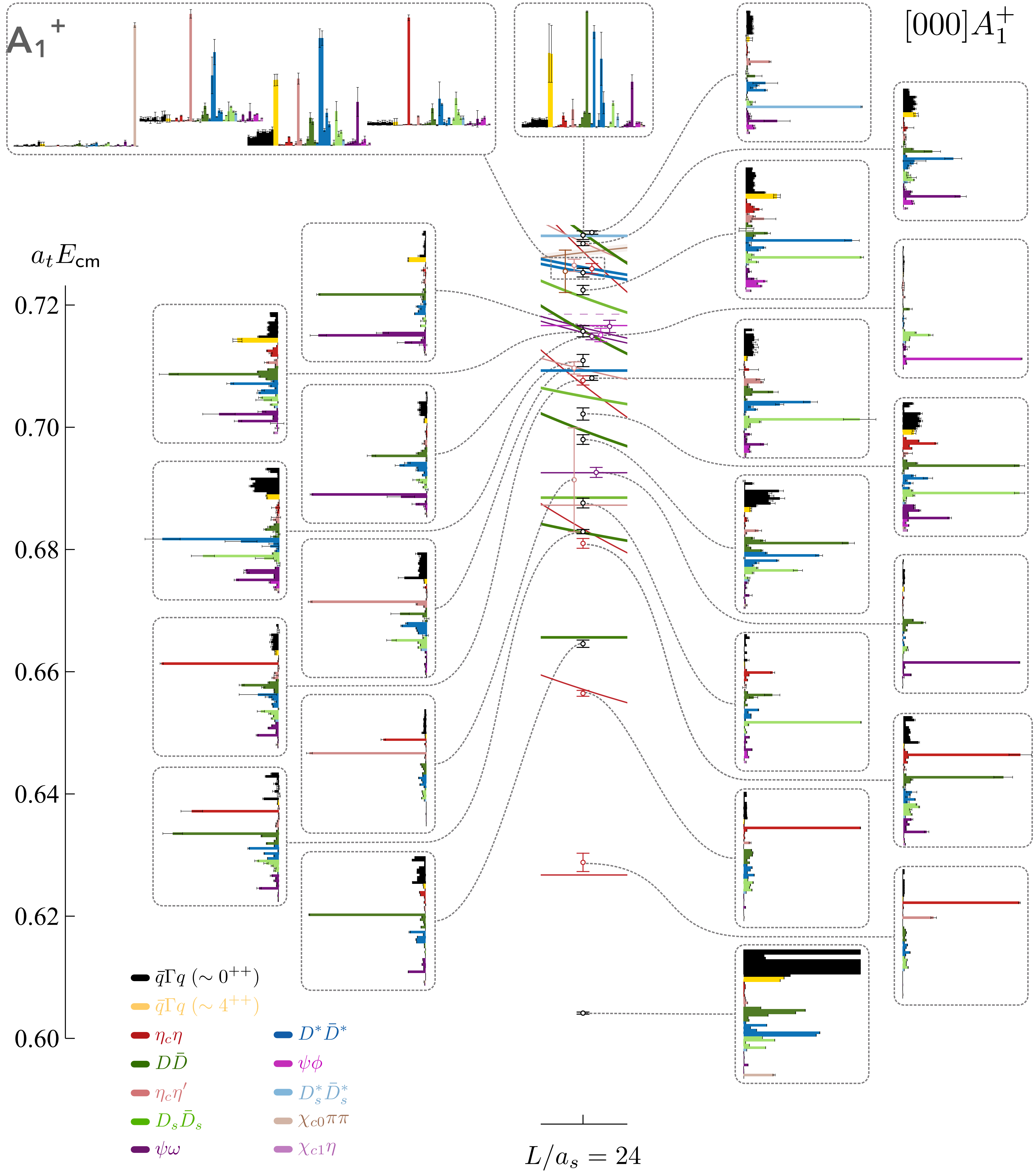
- compute a large correlation matrix
- solve generalised eigenvalue problem to extract energies

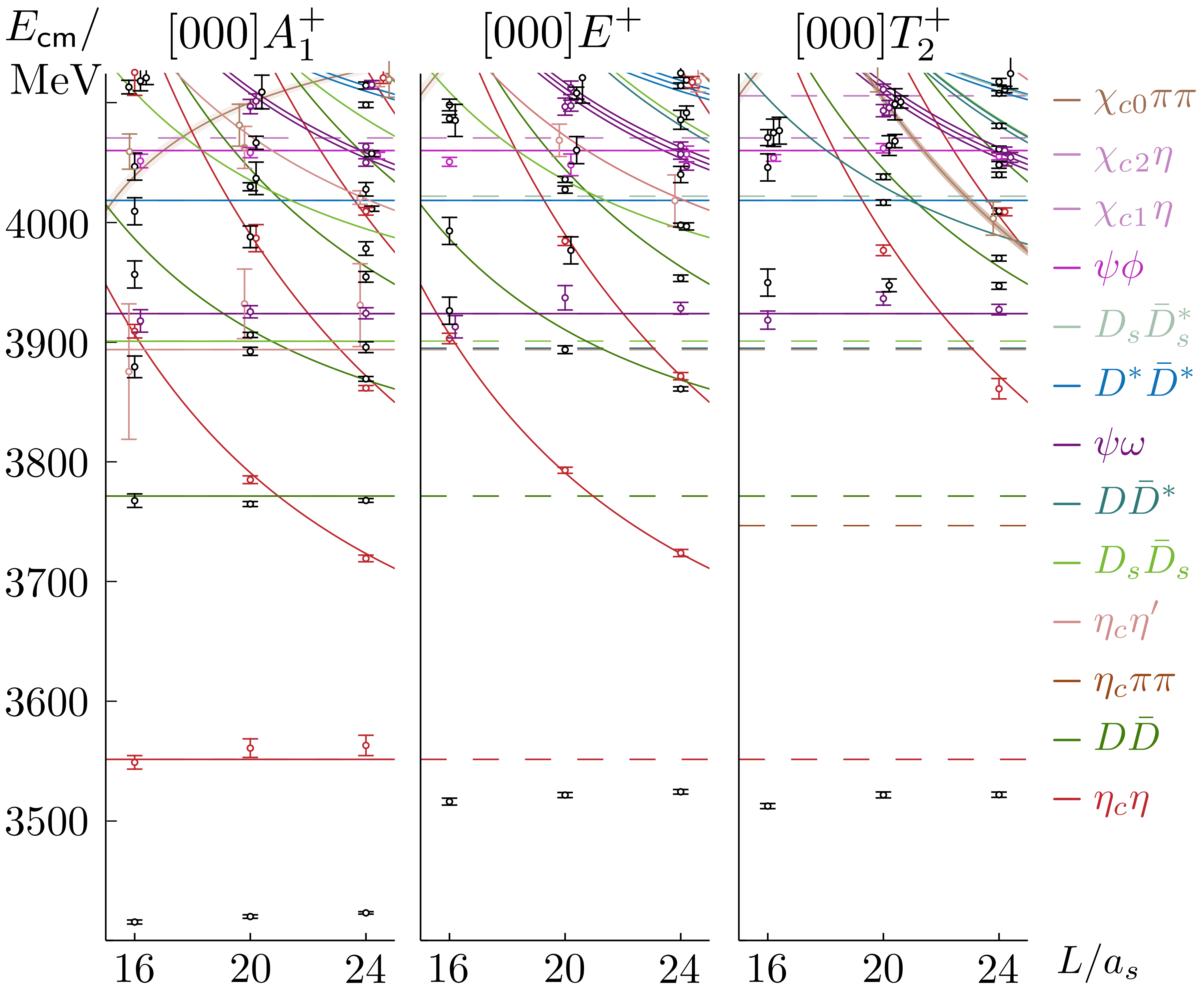


Level counting is completely unclear

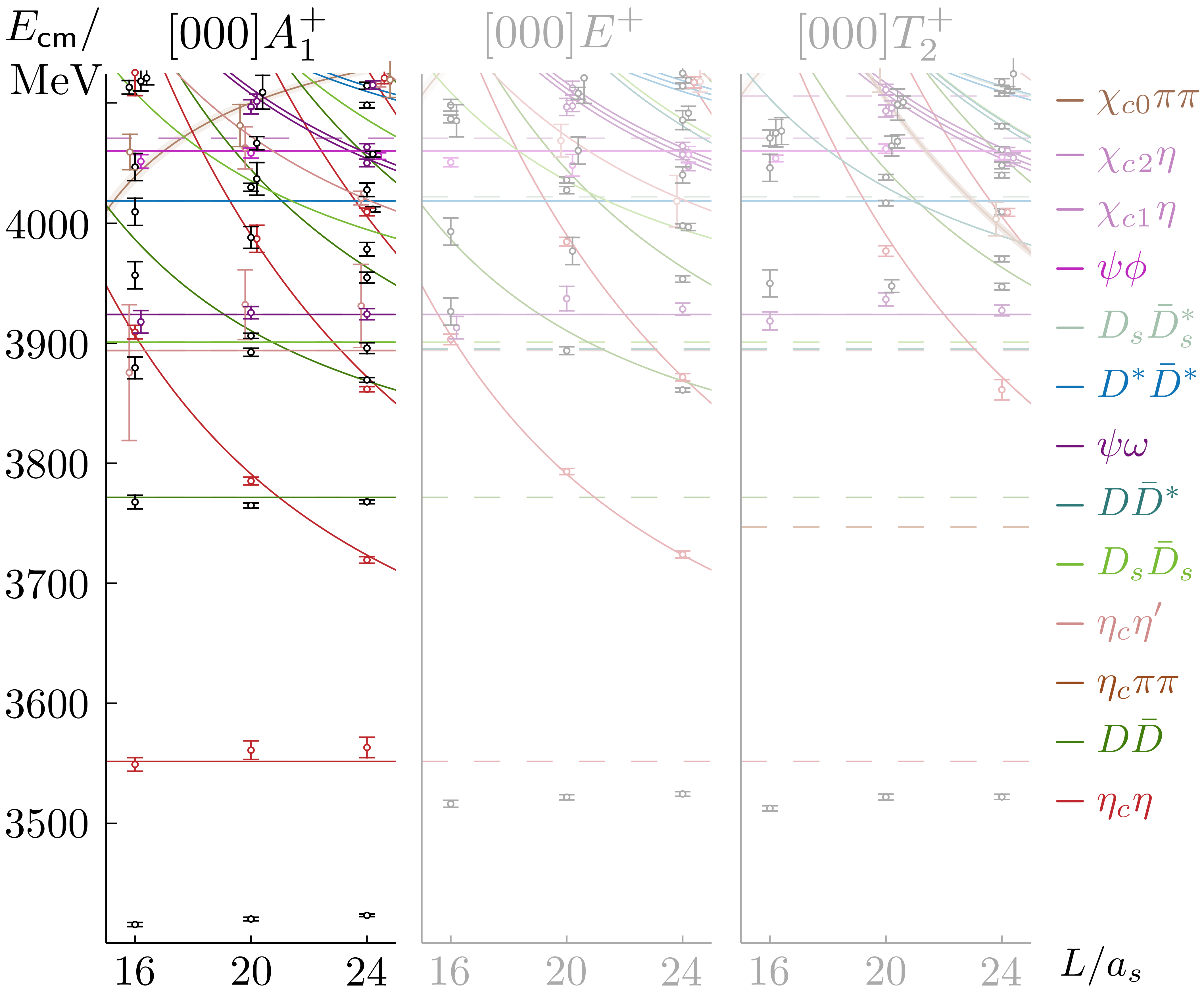
- Near threshold behaviour?
- Multiple decoupled resonances?

Probably one resonance

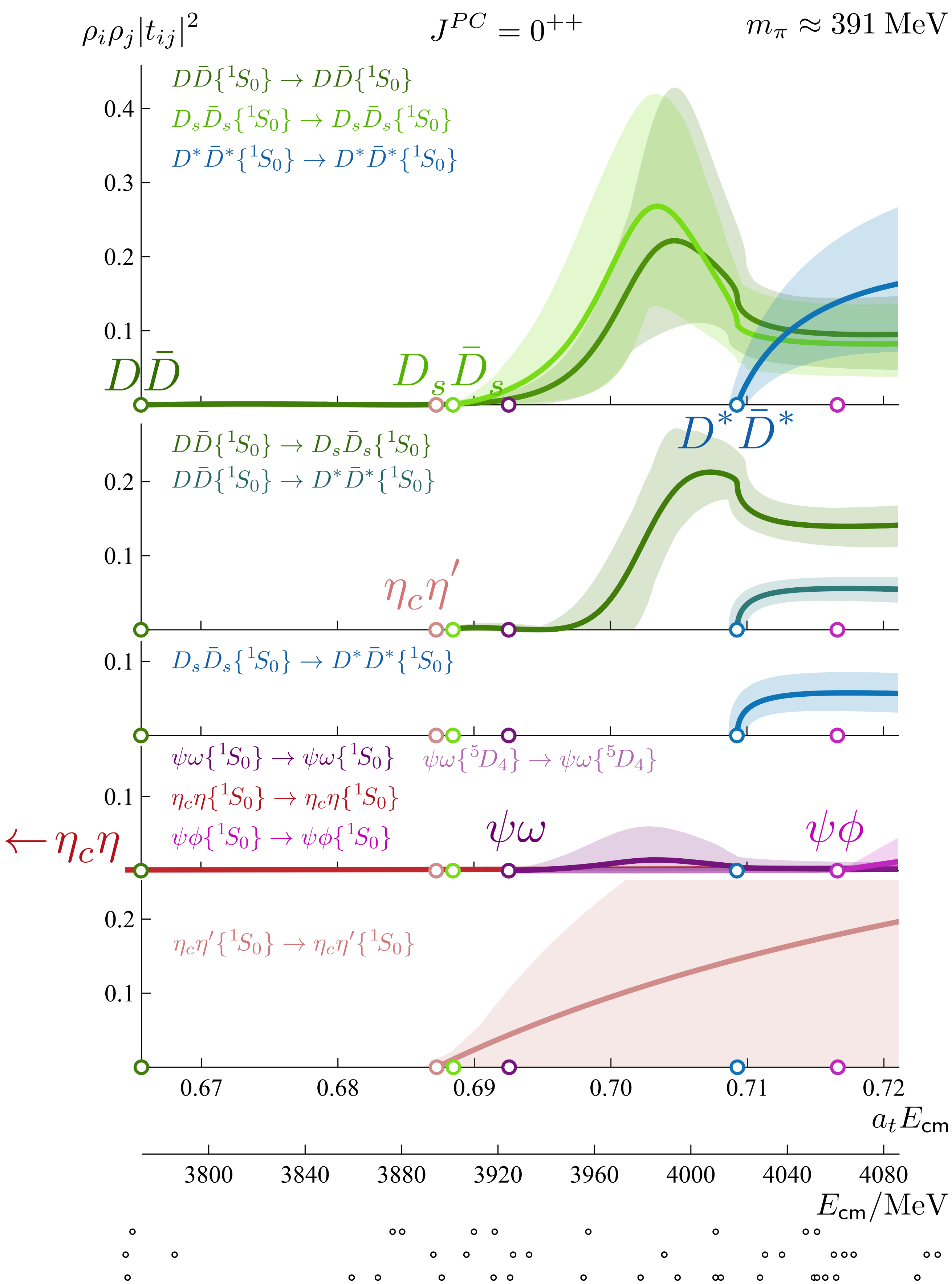

$[000]A_1^+$
$a_t E_{cm}$
0.72
0.70
0.68
0.66
0.64
0.62
0.60
$\bar{q}\Gamma q\ (\sim 0^{++})$
$\bar{q}\Gamma q\ (\sim 4^{++})$
$\eta_c\eta$
$D\bar{D}$
$\eta_c\eta'$
$D_s\bar{D}_s$
$\psi\omega$
$D^*\bar{D}^*$
$\psi\phi$
$D_s^*\bar{D}_s^*$
$\chi_{c0}\pi\pi$
$\chi_{c1}\eta$
$L/a_s = 24$


E_{cm}/ MeV
[000] A_1^+
[000] E^+
[000] T_2^+
4000
3900
3800
3700
3600
3500
16
20
24
L/a_s
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$

"0⁺⁺" "2⁺⁺" "2⁺⁺ & 3⁺⁺"


$E_{\mathsf{cm}}/$ MeV
$[000]A_1^+$
$[000]E^+$
$[000]T_2^+$
3500
3600
3700
3800
3900
4000
16
20
24
L/a_s
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$

"0++" "2++" "2++ & 3++"



three channels open close together: $\eta_c\eta'$, $D_s\bar{D}_s$, $\psi\omega$

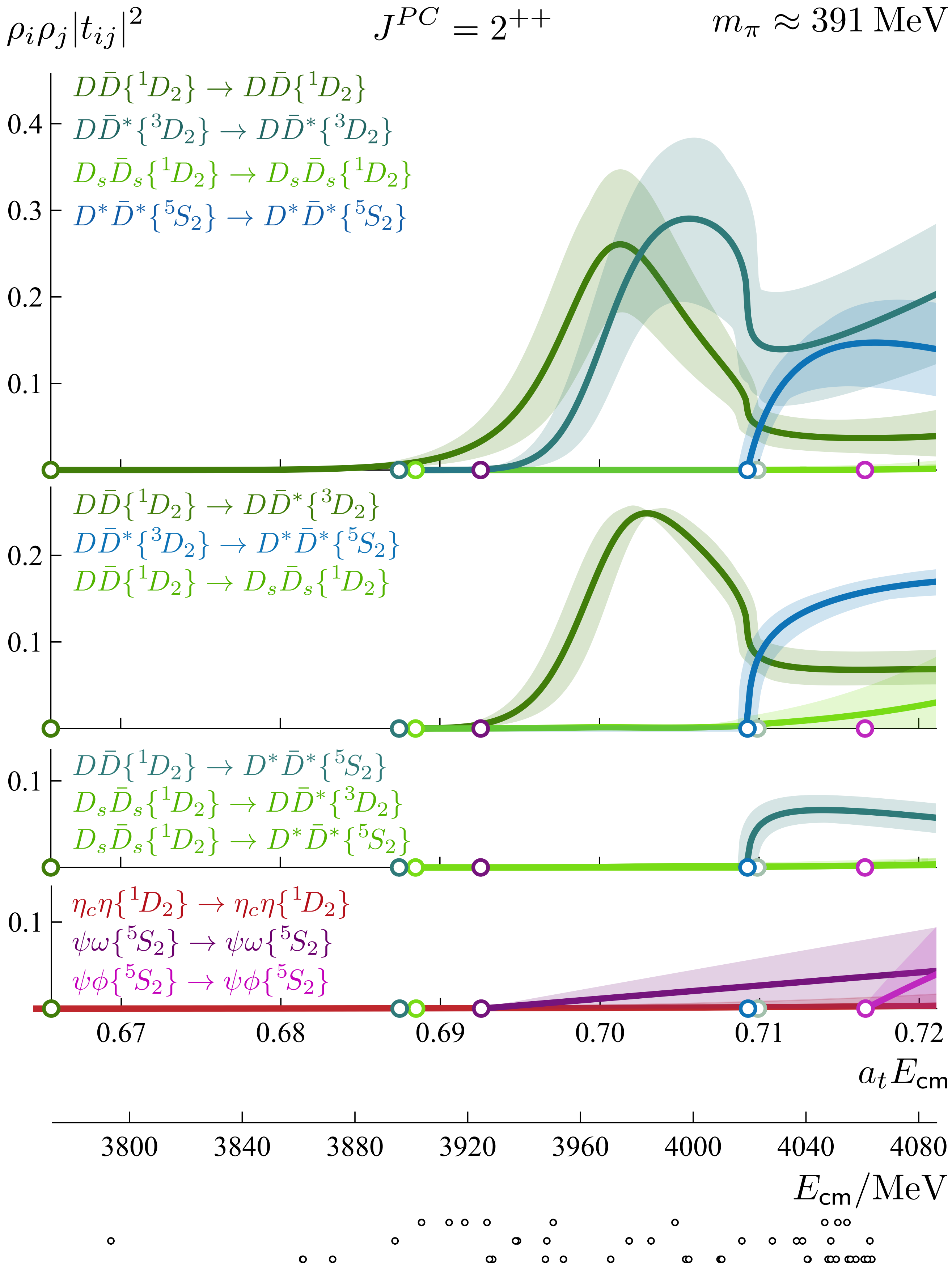
operator overlaps suggest $D^*\bar{D}^*$ is important

$\psi\phi$ has been seen to be important in some places

consider 7-channel system

$$K_{ij} = \frac{g_i g_j}{m^2 - s} + \gamma_{ij}$$

K-matrix pole terms become necessary to obtain a good quality of fit



7-channels, mixture of *S* and *D*

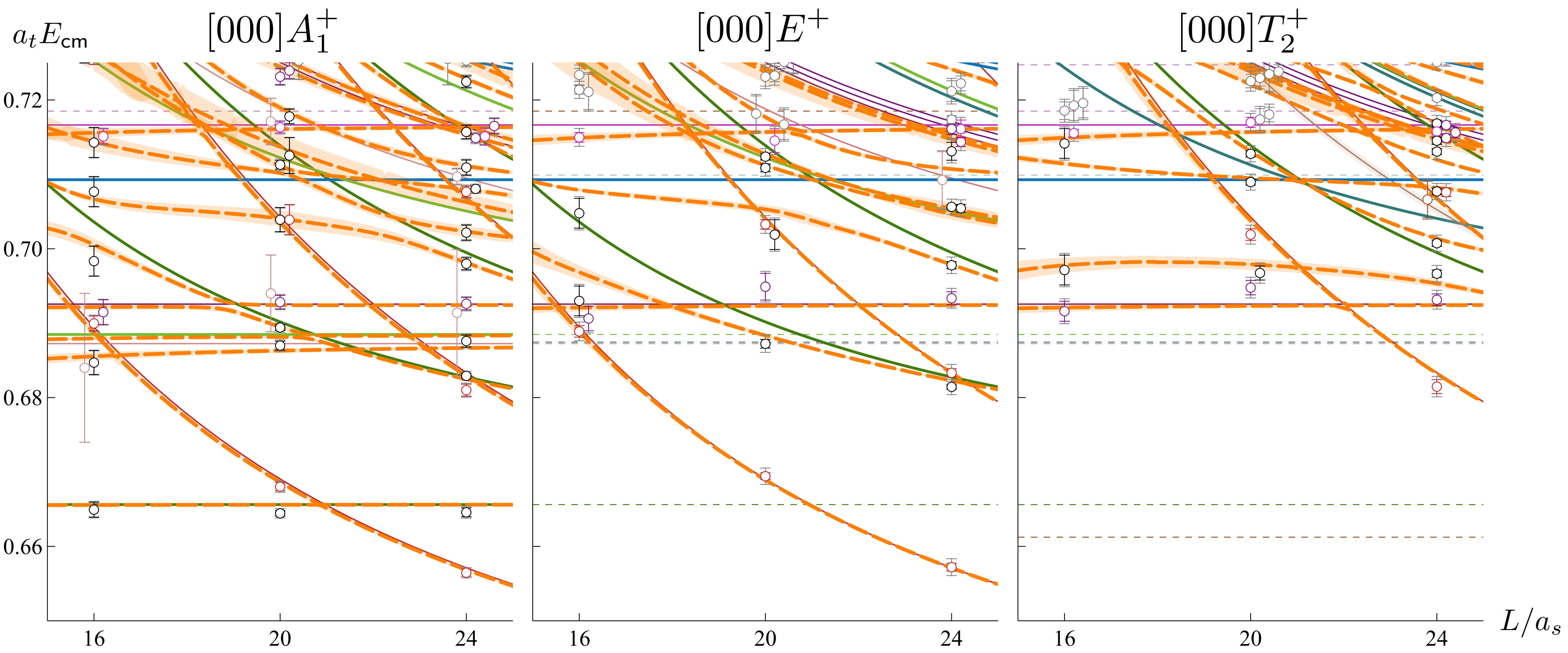
$D\bar{D}, D_s\bar{D}_s\{^1D_2\}$ $D\bar{D}^*\{^3D_2\}$ $D^*\bar{D}^*\{^5S_2\}$

$\eta_c\eta\{^1D_2\}$ $\psi\omega, \psi\phi\{^5S_2\}$

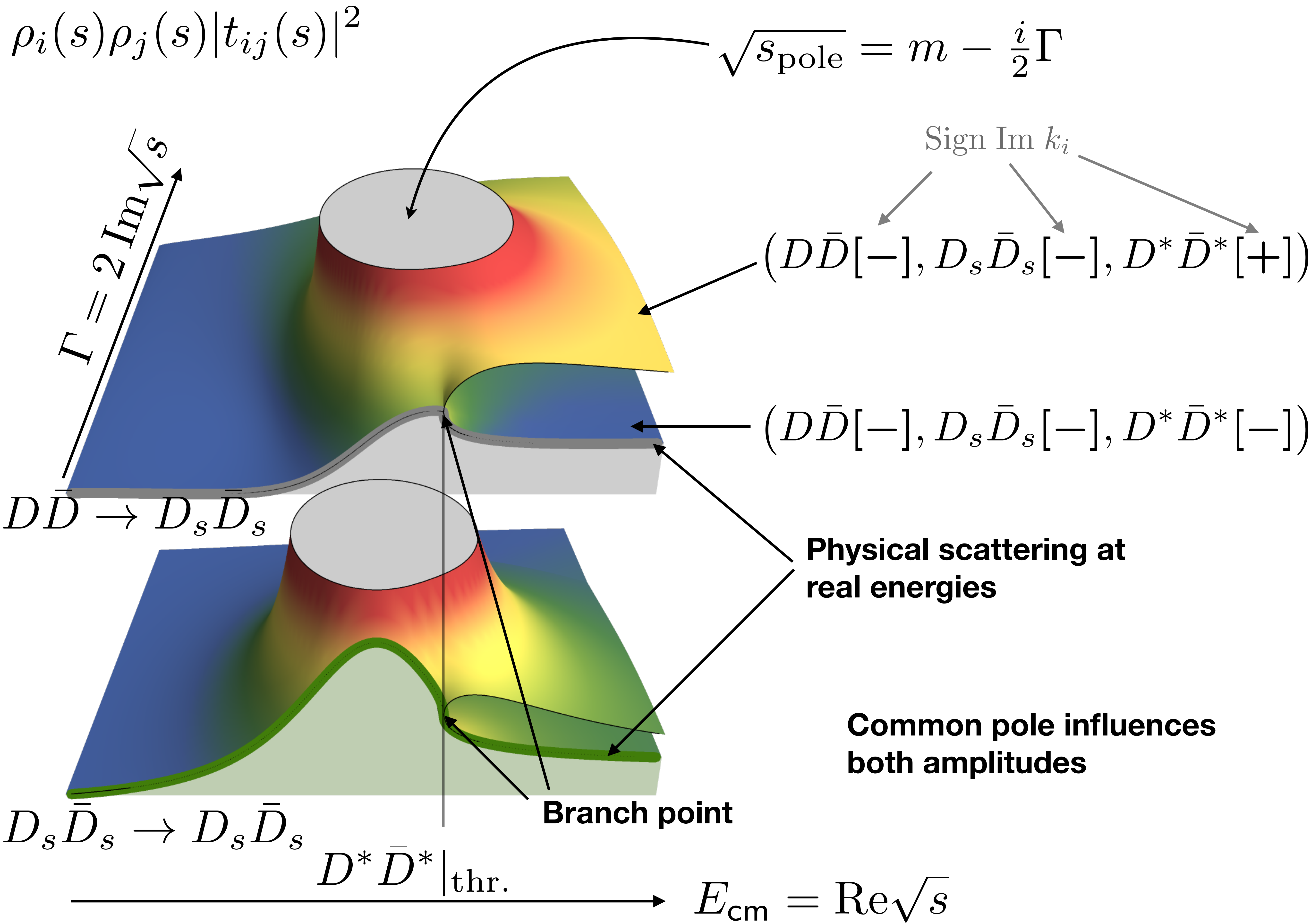
peaks at a similar energy

very small DsDs amplitudes -
some phase space suppression

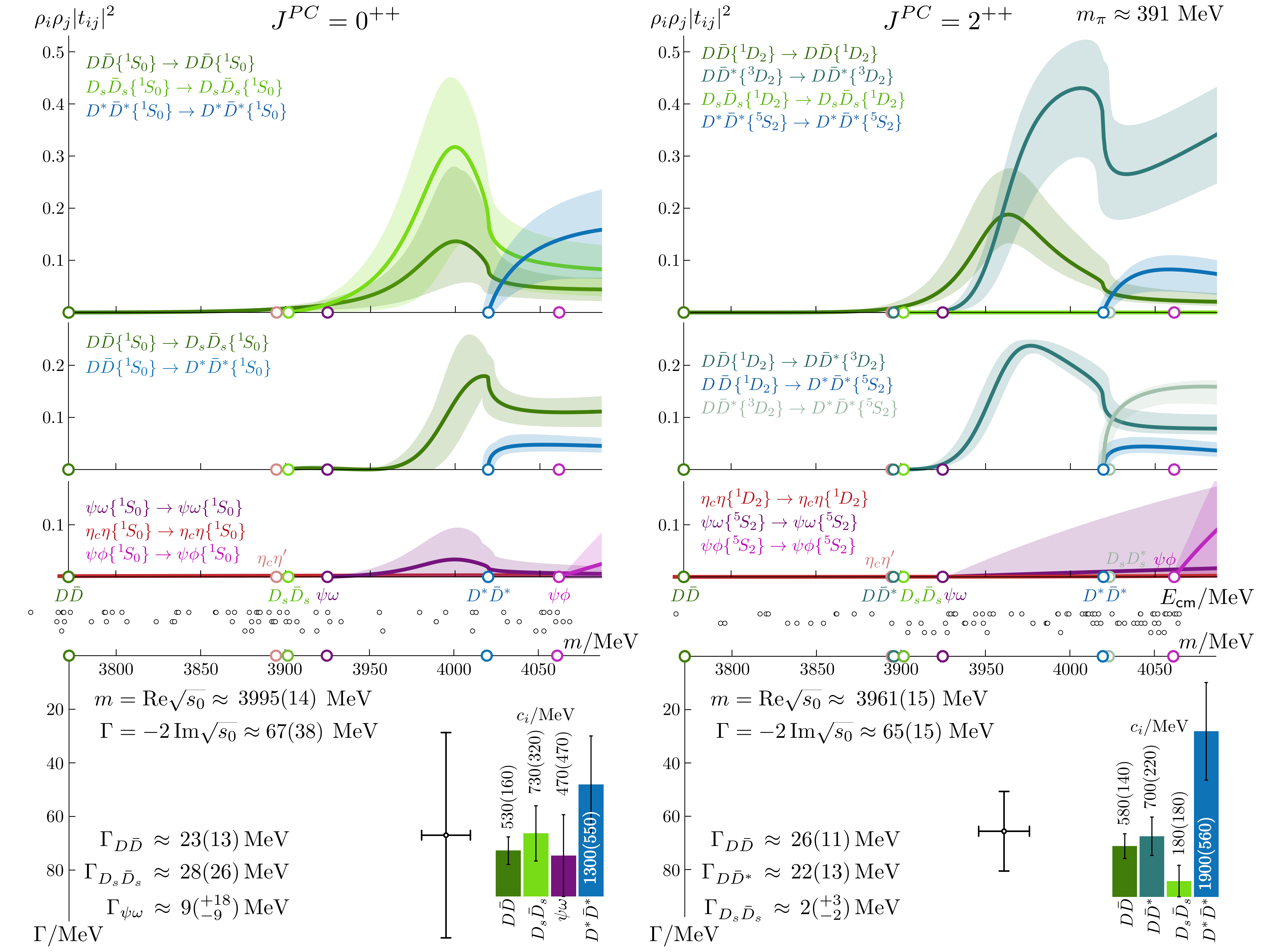
DD* is large -
similar phase space to DsDs



$$\det[\mathbf{1} + i\boldsymbol{\rho}\cdot\boldsymbol{t}\,(\mathbf{1} + i\boldsymbol{\mathcal{M}}(L))] = 0$$



Common pole influences both amplitudes



$\rho_i \rho_j |t_{ij}|^2$
$J^{PC} = 0^{++}$
$J^{PC} = 2^{++}$
$m_\pi \approx 391$ MeV
$D\bar{D}\{^1S_0\} \to D\bar{D}\{^1S_0\}$
$D_s\bar{D}_s\{^1S_0\} \to D_s\bar{D}_s\{^1S_0\}$
$D^*\bar{D}^*\{^1S_0\} \to D^*\bar{D}^*\{^1S_0\}$
$D\bar{D}\{^1S_0\} \to D_s\bar{D}_s\{^1S_0\}$
$D\bar{D}\{^1S_0\} \to D^*\bar{D}^*\{^1S_0\}$
$\psi\omega\{^1S_0\} \to \psi\omega\{^1S_0\}$
$\eta_c\eta\{^1S_0\} \to \eta_c\eta\{^1S_0\}$
$\psi\phi\{^1S_0\} \to \psi\phi\{^1S_0\}$
$\eta_c\eta'$
$D\bar{D}\{^1D_2\} \to D\bar{D}\{^1D_2\}$
$D\bar{D}^*\{^3D_2\} \to D\bar{D}^*\{^3D_2\}$
$D_s\bar{D}_s\{^1D_2\} \to D_s\bar{D}_s\{^1D_2\}$
$D^*\bar{D}^*\{^5S_2\} \to D^*\bar{D}^*\{^5S_2\}$
$D\bar{D}\{^1D_2\} \to D\bar{D}^*\{^3D_2\}$
$D\bar{D}\{^1D_2\} \to D^*\bar{D}^*\{^5S_2\}$
$D\bar{D}^*\{^3D_2\} \to D^*\bar{D}^*\{^5S_2\}$
$\eta_c\eta\{^1D_2\} \to \eta_c\eta\{^1D_2\}$
$\psi\omega\{^5S_2\} \to \psi\omega\{^5S_2\}$
$\psi\phi\{^5S_2\} \to \psi\phi\{^5S_2\}$
$D_sD_s^*$ $\psi\phi$
$D\bar{D}$ $D_s\bar{D}_s$ $\psi\omega$ $D^*\bar{D}^*$ $\psi\phi$
$D\bar{D}$ $D\bar{D}^*$ $D_s\bar{D}_s$ $\psi\omega$ $D^*\bar{D}^*$
E_{cm}/MeV
m/MeV
3800 3850 3900 3950 4000 4050
0.1 0.2 0.3 0.4 0.5
20 40 60 80
Γ/MeV
$m = \mathrm{Re}\sqrt{s_0} \approx 3995(14)$ MeV
$\Gamma = -2\,\mathrm{Im}\sqrt{s_0} \approx 67(38)$ MeV
$\Gamma_{D\bar{D}} \approx 23(13)$ MeV
$\Gamma_{D_s\bar{D}_s} \approx 28(26)$ MeV
$\Gamma_{\psi\omega} \approx 9(^{+18}_{-9})$ MeV
c_i/MeV
530(160) 730(320) 470(470) 1300(550)
$D\bar{D}$ $D_s\bar{D}_s$ $\psi\omega$ $D^*\bar{D}^*$
$m = \mathrm{Re}\sqrt{s_0} \approx 3961(15)$ MeV
$\Gamma = -2\,\mathrm{Im}\sqrt{s_0} \approx 65(15)$ MeV
$\Gamma_{D\bar{D}} \approx 26(11)$ MeV
$\Gamma_{D\bar{D}^*} \approx 22(13)$ MeV
$\Gamma_{D_s\bar{D}_s} \approx 2(^{+3}_{-2})$ MeV
580(140) 700(220) 180(180) 1900(560)
$D\bar{D}$ $D\bar{D}^*$ $D_s\bar{D}_s$ $D^*\bar{D}^*$

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Scalar and tensor charmonium

- at m_π=391 MeV, one scalar and one tensor pole is found.
- The **level counting** is not obviously different from the **quark model**

- large **coupled-channel** effects in OZI **connected D-meson channels**
- OZI **disconnected** channels look **small everywhere**

- we have extracted a **complete** unitary **S-matrix** and this naturally **connects** features seen in **different channels** and simplifies the overall picture
- some amplitudes are **very different** to the simple **Breit-Wigners** often used in experimental analyses

- a clear, as yet unobserved, 3++ resonance is present in DDbar* & a bound state in 2-+
- we **do not find** a **near-threshold DDbar** state (between 3700 and 3860 MeV)

- these methods can also be applied to the X(3872) 1++ channel

Full details and references:
arXiv: 2309.14070 (7 pages)
arXiv: 2309.14071 (55 pages)

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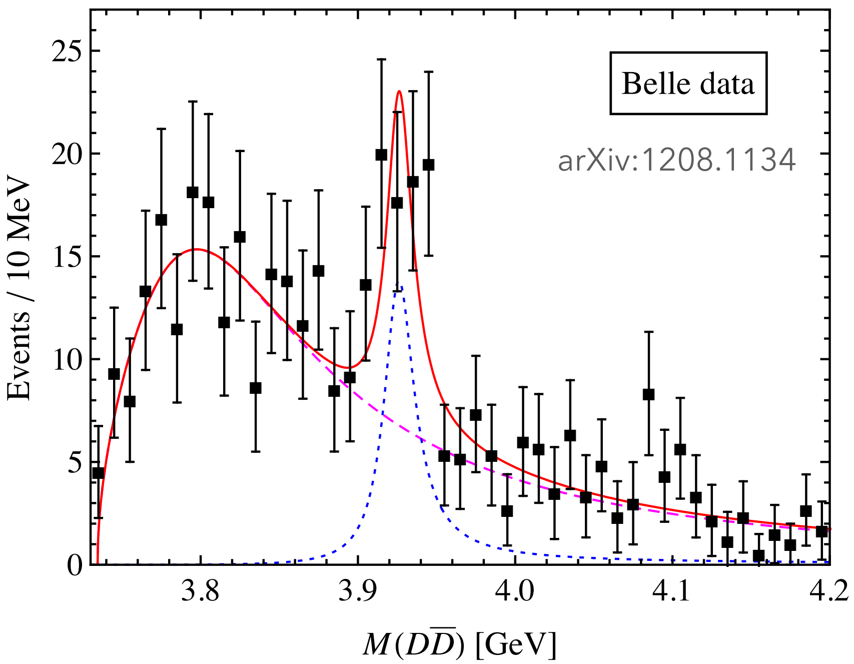
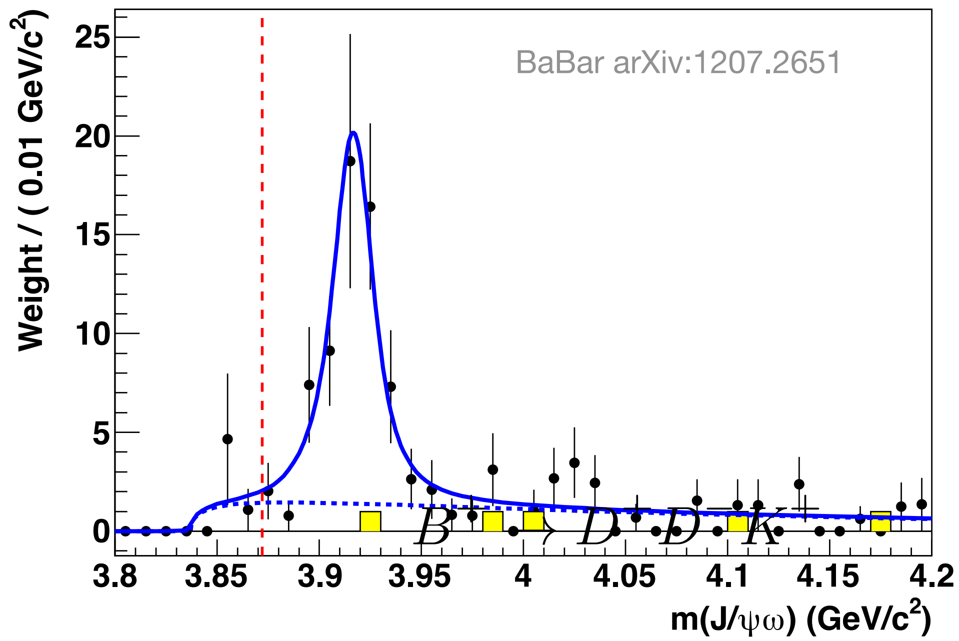
DiRAC + Lattice QCD for hadron spectroscopy

- Large computers and lattice QCD theory are a powerful combination having real world impact on hadron spectroscopy
- It has proven very useful to have clear first-principles theoretical results given the complexity of the experimental data
- DiRAC has been instrumental in enabling these calculations
- Advancement in the last 10 years has been rapid - from simple elastic scattering of spin-0 hadrons to coupled channel scattering involving many channels including hadrons with spin
- Future prospects are bright (see for example Max Hansen's slides from last year)

$\gamma\gamma \to D\bar{D}$



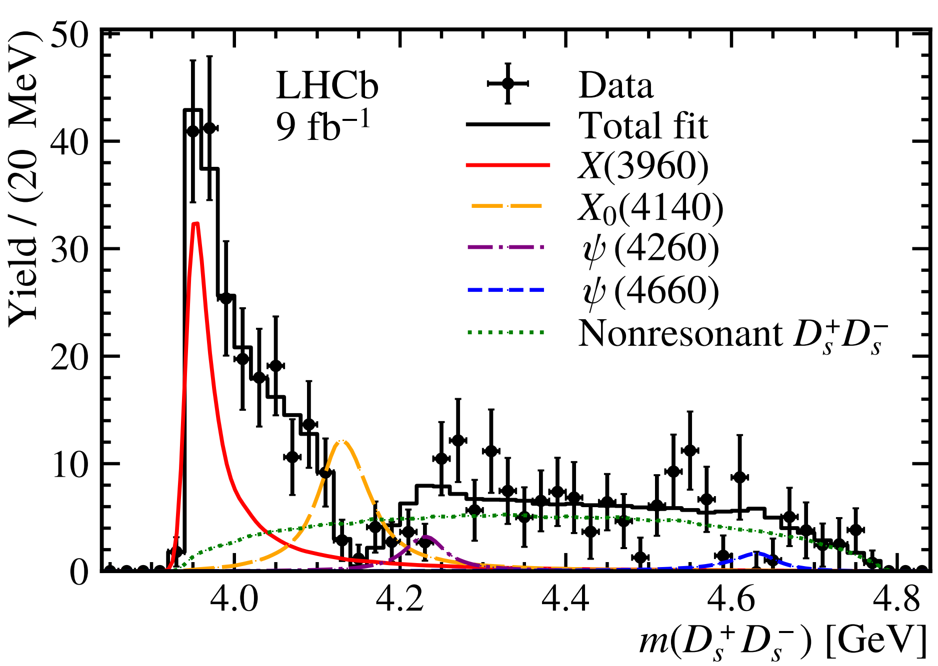
$J/\psi\omega$



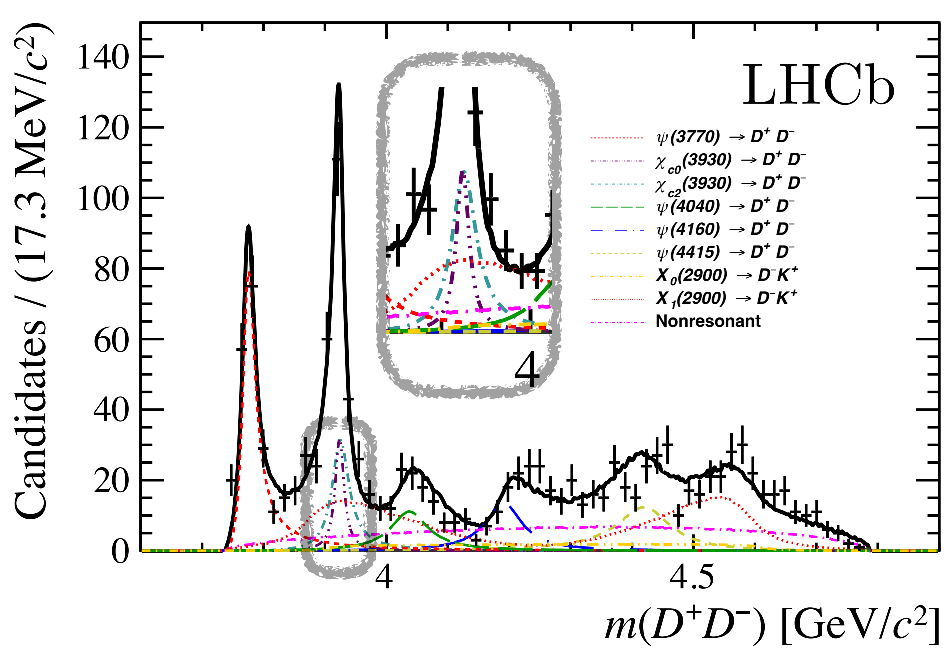
$$m = (3919.4 \pm 2.2 \pm 1.6)\ \mathrm{MeV}$$
$$\Gamma = (13 \pm 6 \pm 3)\ \mathrm{MeV}$$
$$J^P = 0^+$$

Just a few examples
Many many more
(References in the longer paper)

$B^+ \to D_s^+ D_s^- K^+$ arXiv:2210.15153 LHCb



$B^+ \to D^+ D^- K^+$ arXiv:2009.00026



overlapping 0++ and 2++ resonances around 3925 MeV

no need for a low 0++ resonance

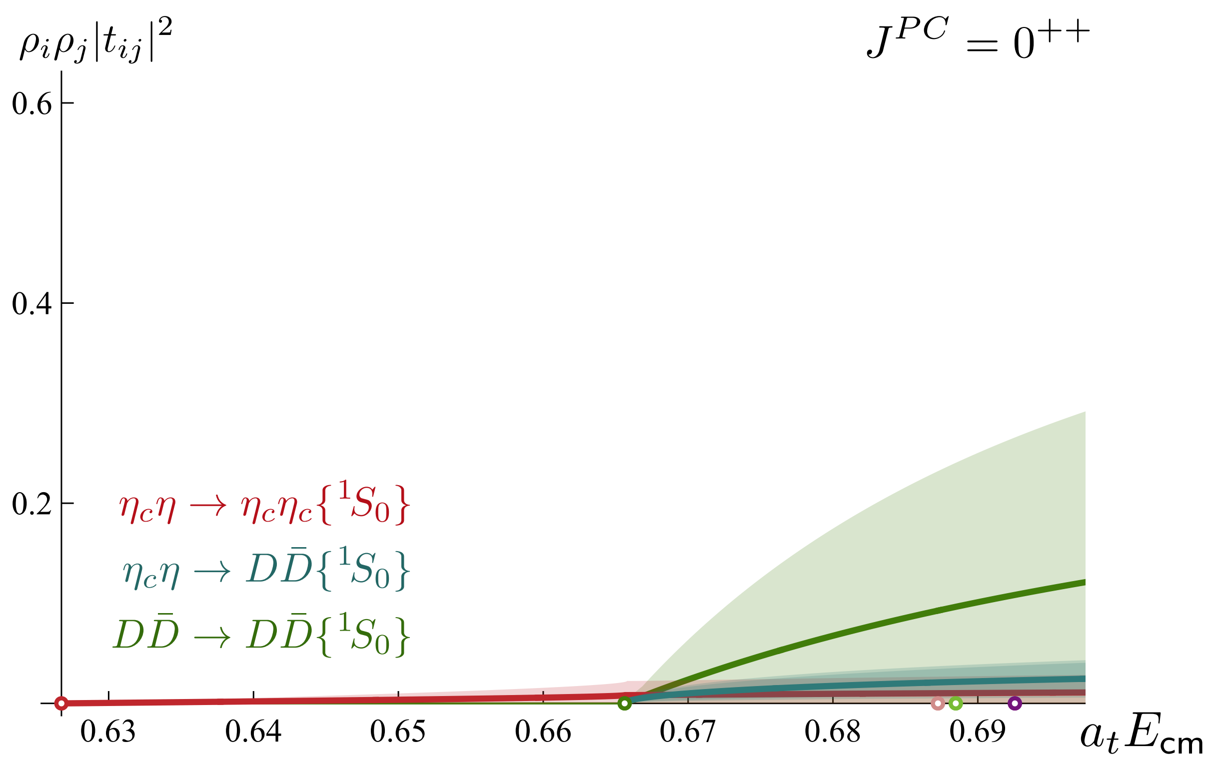
$$\boldsymbol{S} = \boldsymbol{1} + 2i\boldsymbol{\rho}^{\frac{1}{2}} \cdot \boldsymbol{t} \cdot \boldsymbol{\rho}^{\frac{1}{2}}$$

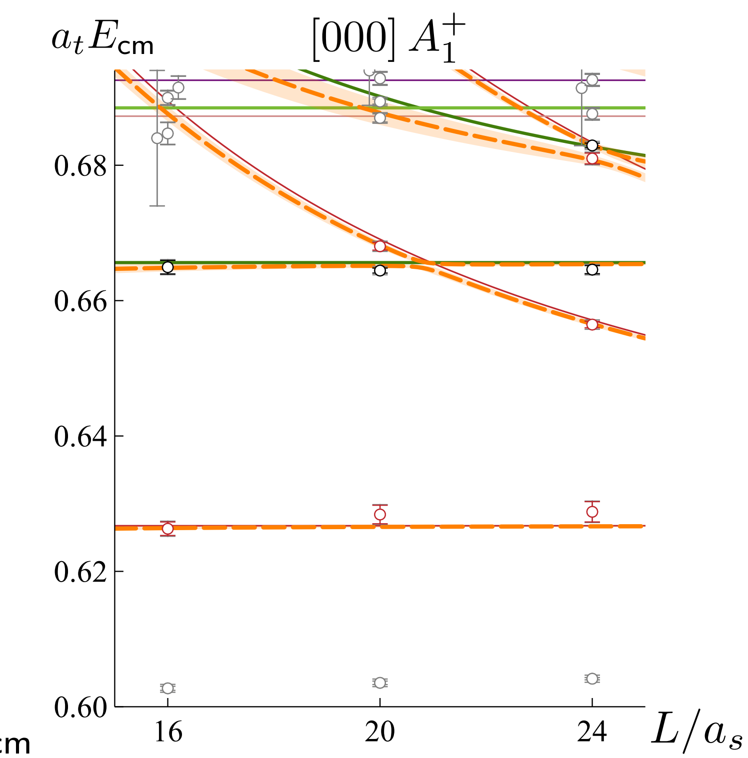
$$\boldsymbol{t}^{-1} = \boldsymbol{K}^{-1} + \boldsymbol{I}$$

$$\mathrm{Im} I_{ij} = -\rho_i = 2k_i/\sqrt{s}$$

$$\det[\boldsymbol{1} + i\boldsymbol{\rho} \cdot \boldsymbol{t}\,(\boldsymbol{1} + i\boldsymbol{\mathcal{M}}(L))] = 0$$

$$\boldsymbol{K} = \begin{bmatrix} \gamma_{\eta_c\eta\to\eta_c\eta} & \gamma_{\eta_c\eta\to D\bar{D}} \\ \gamma_{\eta_c\eta\to D\bar{D}} & \gamma_{D\bar{D}\to D\bar{D}} \end{bmatrix}$$

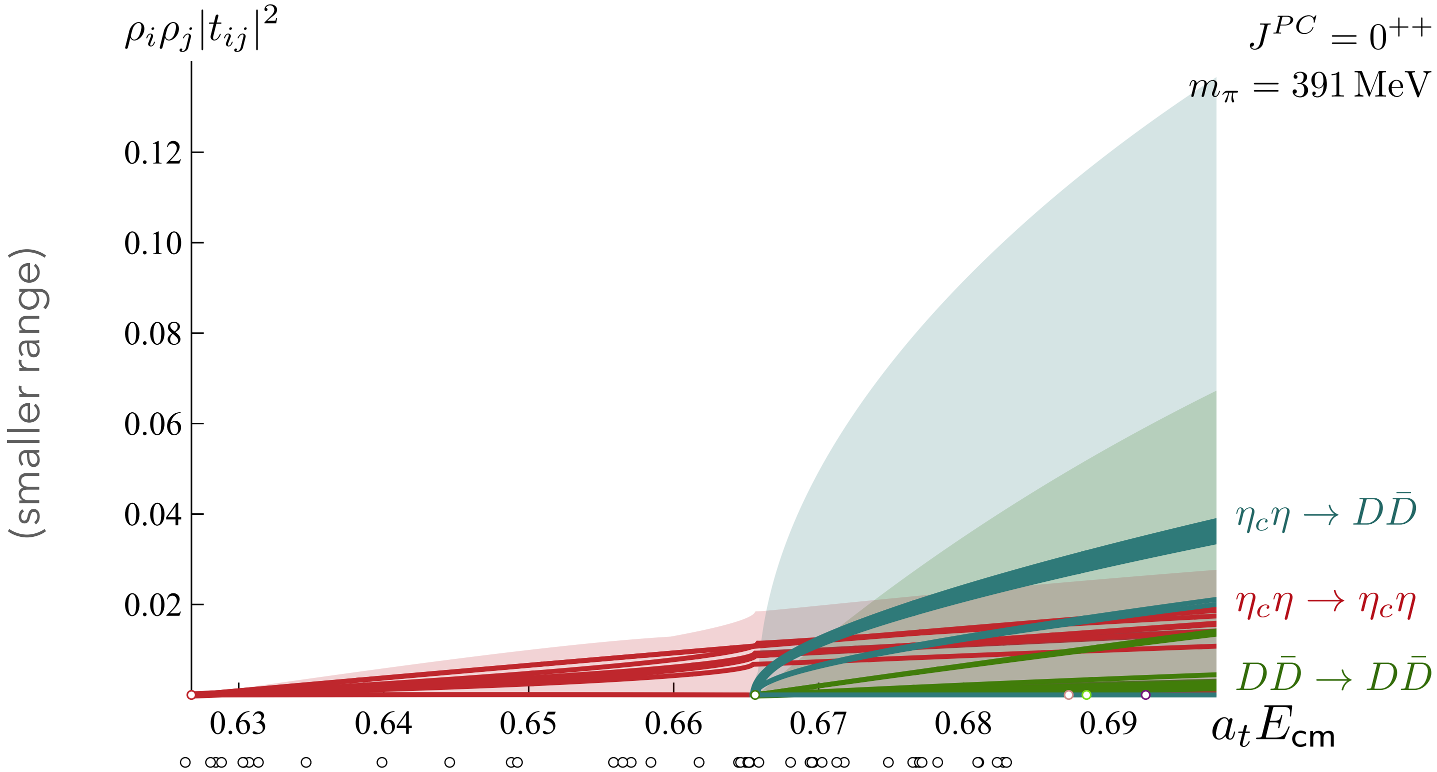




using rest-frame only

$$\begin{aligned} \gamma_{\eta_c\eta\to\eta_c\eta} &= (0.34 \pm 0.23 \pm 0.09) \\ \gamma_{\eta_c\eta\to D\bar{D}} &= (0.58 \pm 0.29 \pm 0.05) \\ \gamma_{D\bar{D}\to D\bar{D}} &= (1.39 \pm 1.19 \pm 0.24) \end{aligned} \qquad \begin{bmatrix} 1.00 & 0.77 & -0.24 \\ & 1.00 & -0.22 \\ & & 1.00 \end{bmatrix}$$

$$\chi^2/N_{\mathrm{dof}} = \frac{5.65}{10-3} = 0.81$$



using zero and non-zero total momentum