Standard Model precision tests from lattice QCD

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- Divide matter into its smallest constituents

$$\begin{aligned} 10^{-10}\,\mathrm{m}: &\quad \text{Atom} \\ 10^{-14}\,\mathrm{m}: &\quad \text{Nucleus (protons/neutrons)} \\ 10^{-19}\,\mathrm{m}: &\quad \text{Elementary particles} \end{aligned}$$

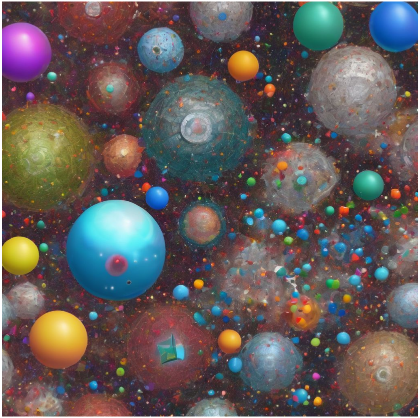


Figure from img2go

- Particle content
- Interact via forces

  Strong nuclear force

  Weak nuclear force

  Electromagnetic force

  ~~Gravity~~
- What about real world?

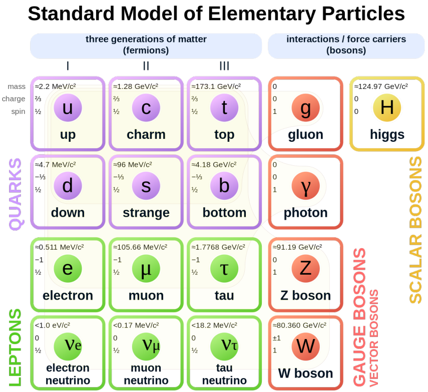
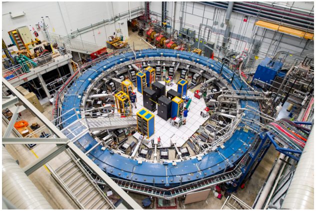


Figure from Wikipedia

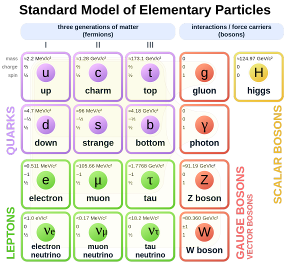
- $\sim$ 20 parameters
- Make predictions!
- Works exceptionally well
- Does not explain:
  Dark matter/energy
  Matter asymmetry
  ...

$\Longrightarrow$ Need to test the Standard Model to find what is beyond



Experiment

$=$



Theory prediction

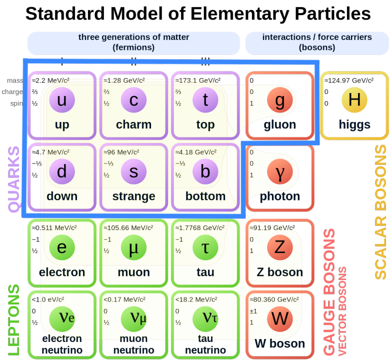
$\underbrace{+ \quad ?}_{\textit{Unknown}}$

Searching for new particles with precision calculations

$\longrightarrow$ If real tension found, look for new particles in dedicated experiments

$\longrightarrow$ If no tension, we still constrain new physics

Strong force = Quantum ChromoDynamics



- Quarks and gluons not free particles
- Bound into protons, neutrons, pions, kaons, ...
- Important at low energies
- How to do QCD calculations?
  - Dispersion theory
  - Effective field theory
  - Lattice QCD

Effective field theory for QCD

- Relevant degrees of freedom: Pions, kaons and eta
- Symmetries of QCD. Systematic power counting: LO, NLO, ...
- Systematically improve precision: higher order
- Everything in terms of low-energy constants
- Bottleneck: Constants must be known for new predictions
- Despite this, technique very useful for lattice QCD

	# LECs
LO	2
NLO	12
NNLO	94
NNNLO	1254

[Wess, Zumino 71; Witten 83; Gasser, Leutwyler 83/84; Bijnens, Colangelo, Ecker 99; Bijnens, Girlanda, Talavera 02; Bijnens, NHT, Wang 18; Bijnens, NHT, Ruiz-Vidal 23]

Lattice QCD

- Simulate QCD on the computer: Monte-Carlo methods
- Discretise space and time onto grid of finite extent

$$\langle \mathcal{O} \rangle = \frac{\int DU \, e^{-S[U]} \, \mathcal{O}[U]}{\int DU \, e^{-S[U]}}$$

- Action $S[U]$ contains the underlying physics of QCD
- Statistical/systematic uncertainties, e.g.

- Discretisation effects: a
- Finite-volume effects: L
- Physics: $L \to \infty$, $a \to 0$, ...

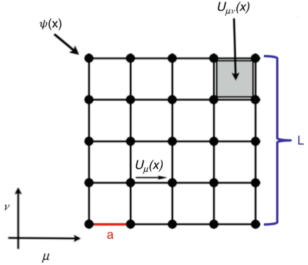


Figure from Ratti et al., 2021

- Lattice QCD: Systematically improvable with three ingredients

⋆ Computer power:

DiRAC

⋆ Algorithmic/software development

⋆ Formal/data analysis developments: Effective theory techniques

- Next: look at some state-of-the art precision tests (biased)

Magnetic moment of the muon

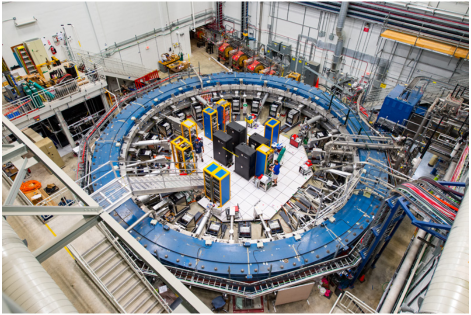
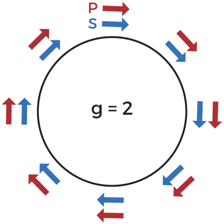


Figure from Fermilab

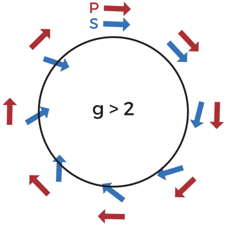
- Muon has spin **S** and magnetic moment

$$\vec{\mu} = g_\mu \frac{Q_\mu e}{2m_\mu} \vec{S}\,, \qquad a_\mu = \frac{(g-2)_\mu}{2}$$

- $g_\mu = 2$ is the classical value for gyromagnetic ratio
- Spin interacts with external magnetic field
- Let muon move circularly in homogeneous magnetic field



Not real life



Anomalous [Schwinger 1948]

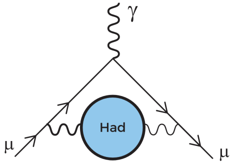
- Standard Model [White Paper, 2020] vs. Experimental value Brookhaven/Fermilab

$$a_\mu^{\mathrm{SM},\,2020} = 116\,591\,810(43) \times 10^{-11}\,,$$
$$a_\mu^{\mathrm{exp}} = 116\,592\,059(22) \times 10^{-11}\,, \qquad \longleftarrow 0.1 \text{ ppm precision}$$
$$\Delta a_\mu = 249(48) \times 10^{-11}$$

- 5.2σ deviation between experiment and theory [White Paper 2020; Fermilab 2023]

Did we discover new physics!?

- Not really... Lattice QCD does not agree with dispersion theory [BMW 20/24]



Hadronic vacuum polarisation

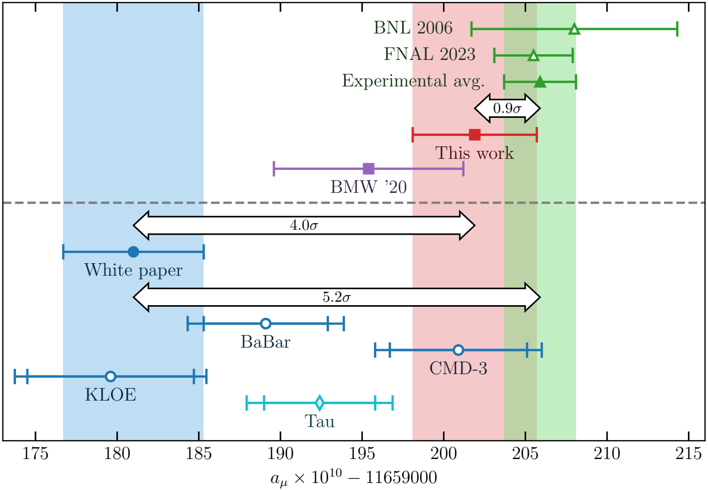


Figure from [BMW 24]

- Theoretical discrepancy: dispersion theory vs lattice QCD
- Lattice result needed scrutiny: A lot of interest from collaborations

The RBC & UKQCD collaborations

Boston University
Nobuyuki Matsumoto

BNL and BNL/RBRC
Peter Boyle
Taku Izubuchi
Christopher Kelly
Shigemi Ohta (KEK)
Amarji Soni
Masaaki Tomii
Xin-Yu Tuo
Shuhei Yamamoto

University of Cambridge
Nelson Lachini

CERN
Matteo Di Carlo
Felix Erben
Andreas Jüttner (Southampton)
Tobias Tsang

Columbia University
Norman Christ
Sarah Fields
Ceran Hu
Yikai Huo
Joseph Karpie (JLab)
Erik Lundstrum
Bob Mawhinney
Bigeng Wang (Kentucky)

University of Connecticut
Tom Blum
Jonas Hildebrand

Luchang Jin
Vaishakhi Moningi
Anton Shcherbakov
Douglas Stewart
Joshua Swaim

DESY Zeuthen
Raoul Hodgson

Edinburgh University
Luigi Del Debbio
Vera Gülpers
Maxwell T. Hansen
Nils Hermansson-Truedsson
Ryan Hill
Antonin Portelli
Azusa Yamaguchi

Johannes Gutenberg University of Mainz
Alessandro Barone

Liverpool Hope/Uni. of Liverpool
Nicolas Garron

LLNL
Aaron Meyer

Autonomous University of Madrid
Nikolai Husung

University of Milano Bicocca
Mattia Bruno

Nara Women's University
Hiroshi Ohki

Peking University
Xu Feng
Tian Lin

University of Regensburg
Andreas Hackl
Daniel Knüttel
Christoph Lehner
Sebastian Spiegel

RIKEN CCS
Yasumichi Aoki

University of Siegen
Matthew Black
Anastasia Boushmelev
Oliver Witzel

University of Southampton
Bipasha Chakraborty
Ahmed Elgaziari
Jonathan Flynn
Joe McKeon
Rajnandini Mukherjee
Callum Radley-Scott
Chris Sachrajda

Stony Brook University
Fangcheng He
Sergey Syritsyn (RBRC)

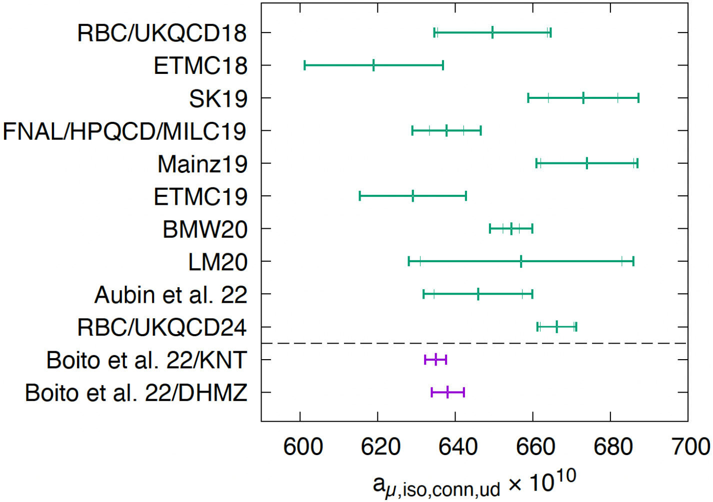


Figure from [RBC/UKQCD(NHT) 24]

- Also other lattice collaborations find the same
- Lattice QCD: No new physics in full $g-2$ [BMW 20/24; Mainz 24]

Flavour physics sector

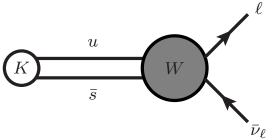
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

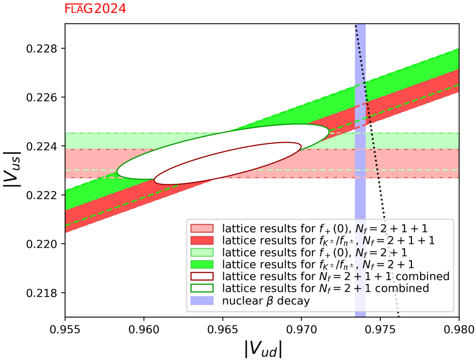
Fundamental parameters of the Standard Model: Essential

A. Complex phase: Matter-antimatter asymmetry of the Universe

B. Search for new physics by testing unitarity relation:

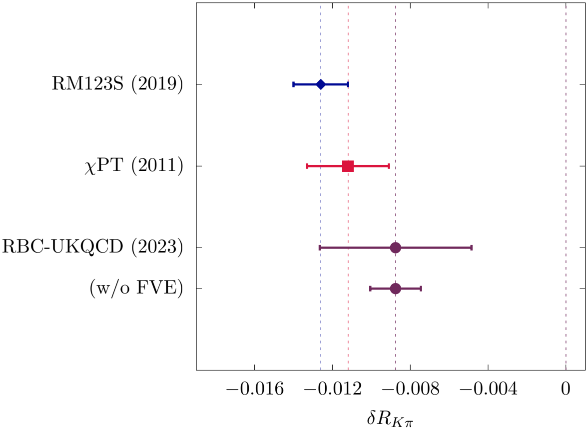
$$1 \stackrel{\mathrm{SM}}{=} |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2$$





- Determine $|V_{ud}|$ and $|V_{us}|$
- Tensions with unitarity?
- Depends on the input!
- Need to better control theory/exp. uncertainties

- $|V_{us}/V_{ud}|$ from leptonic decays of pions and kaons
- Precision goal: (Sub-)% level
- Electromagnetism and quark mass differences essential
- Studied in [RM123S 19; RBC/UKQCD(NHT) 23]



RBC-UKQCD(NHT):

$\delta R_{K\pi} = -0.0086(39)$

total	(39)
total (w/o FVE)	(13)
statistical	(3)
FVE	(37)
fit	(11)
QED quenching	(5)
discretisation	(5)

χPT : $\delta R_{K\pi} = -0.0112(21)$

RM123S 19: $\delta R_{K\pi} = -0.0126(14)$

- ⋆ Improvement 1: Finite-volume effects
- ⋆ Improvement 2: Disconnected diagrams

Improvement 1: Finite-volume effects

- Generic effective field theory to derive volume dependence:

$$Y(L) = \underbrace{Y_0 + Y_{\log} \log(m_P L) + \frac{Y_1}{m_P L}}_{\text{[RM123/S 16]}} \underbrace{+ \frac{Y_2}{(m_P L)^2} + \frac{Y_3}{(m_P L)^3}}_{\text{[Di Carlo, Hansen, NHT, Portelli 22/23]}} + \ldots$$

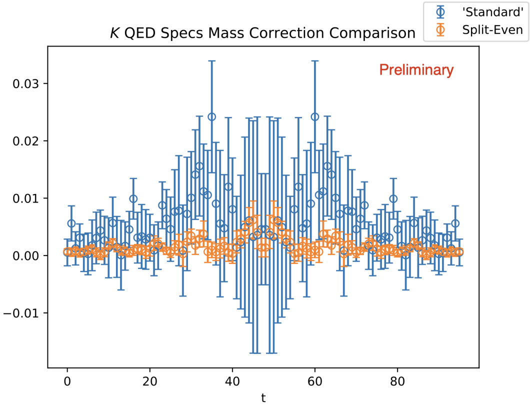
- Bottleneck: Y_3 not fully known in $\mathrm{QED_L}$ and potentially large
- Workaround: New finite-volume electromagnetism
- $\mathrm{QED_r}$ [Davoudi et al. 19; Di Carlo, Hansen, NHT, Portelli In prep]

$$\overline{Y}(L) = \overline{Y}_0 + \overline{Y}_{\log} \log(m_P L) + \frac{\overline{Y}_1}{m_P L} + \frac{\overline{Y}_2}{(m_P L)^2} + \underbrace{\frac{\overline{Y}_3}{(m_P L)^3}}_{=0} + \ldots$$

- Future use: Electromagnetic corrections in Edinburgh/CERN and generally applicable

Improvement 2: Disconnected diagrams

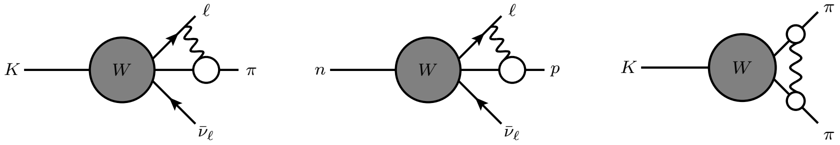
- Disconnected diagrams: General bottleneck for lattice calculations
- Notoriously noisy: Algorithmic efforts in Edinburgh [Harris, Hill, Portelli]



- Finite-volume effects in full vs. connected-only theory? [Hansen, NHT, Ungar]

What else lies in the future?

- $|V_{ud}|$ and $|V_{us}|$: Beta decays of kaons and neutrons
- Matter vs antimatter: $K \to \pi\pi$



- Similar problem for all: Final-state electromagnetic interaction
- Difficult to extract physical information from simulations
- Formal developments: Effective theory techniques and amplitudes

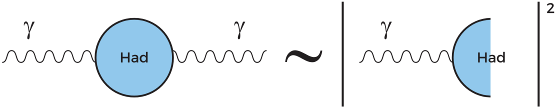
Conclusions and outlook

- Searching for/constraining new physics with precision calculations
- Need to understand potential tensions
- Muon $g-2$: Success story for Lattice QCD
- Lattice QCD will be an essential tool in the future
- Needs formal and algorithmic developments
- Very important: Computational DiRAC resources

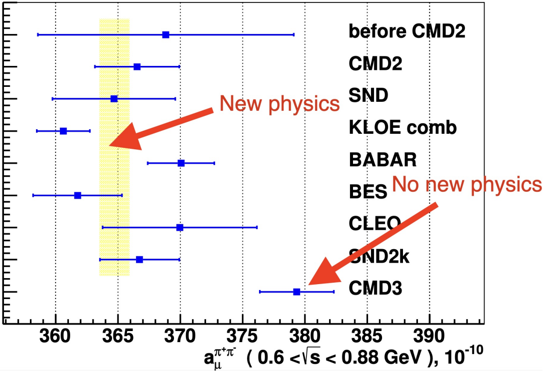
Backup slides

Dispersion theory

- Use experimental data: $e^+e^- \to \gamma^* \to \mathrm{hadrons}$

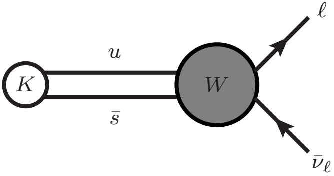


- Muon magnetic moment: Inconsistent data Figure from [CMD3 23]



- Traditional method
- Bottleneck: Need experimental data

- Access $|V_{us}|/|V_{ud}|$ from leptonic kaon/pion decays



$$\underbrace{\Gamma\left(K^- \to \mu^- \nu_\mu\right)}_{\text{Exp.}} \propto |V_{us}|^2 \underbrace{\frac{\left(m_K^2 - m_\mu^2\right)^2}{m_K^3}}_{\text{Exp.}} \underbrace{f_K^2\left(1+\delta R_K\right)}_{\text{Theory}}$$

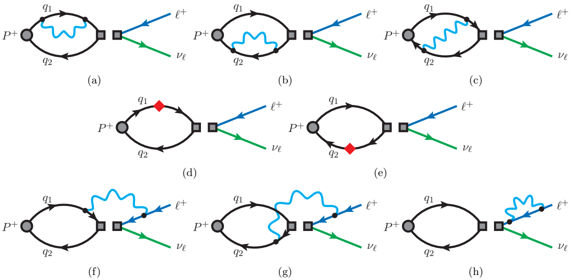
- Combine experiment and theory (lattice)

$$\frac{|V_{us}|^2}{|V_{ud}|^2} = \frac{\text{Kaon exp.}}{\text{Pion exp.}} \times \frac{f_K^2}{f_\pi^2}\left(1 + \delta R_K - \delta R_\pi\right)$$

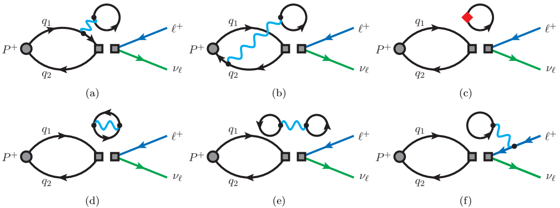
- Isospin-breaking corrections in $\delta R_K - \delta R_\pi$: % level precision

[RM123S 2019; Di Carlo, Hansen, NHT, Portelli 2022; RBC/UKQCD(NHT) 2023]

- Included: Connected



- Excluded: Disconnected



- Correct data with $Y(L)$: $\delta R_P = \delta R_P(L) - Y(L)$

$$Y(L) = \underbrace{Y_0 + Y_{\log}\,\log(m_P L) + \frac{Y_1}{m_P L}}_{\text{[RM123/S 16]}} \quad \underbrace{+\frac{Y_2}{(m_P L)^2} + \frac{Y_3}{(m_P L)^3}}_{\text{[Di Carlo, Hansen, NHT, Portelli 22/23]}} \quad + \ldots$$

- Structure dependent: $1/L^2 + 1/L^3$

$$Y_3 = \frac{32\pi^2\, m_P}{f_P(1-r_\ell^4)}\left\{ c_0(\mathbf{v}_\ell)\,\left[F_V^P - F_A^P + 2m_P^2 r_\ell^2\, F_A^{P\,\prime}\right] + c_0\,\mathcal{C}_\ell \right\}$$

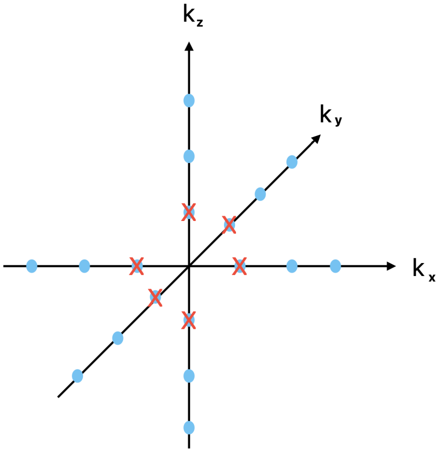
- F_V^P, F_A^P, $F_A^{P\,\prime}$ Lattice [RM-123/S 20/22/23, RBC/UKQCD 23], ChPT [Bijnens et al. 92]
- Cannot determine structure dependent $\mathcal{C}_\ell$: Drove uncertainty
- Known finite-volume coefficients $c_j(\mathbf{v}_\ell)$ at every order
- $\mathrm{QED_r}$: A new finite-volume QED $\overline{c}_0 = 0$ [Di Carlo, Hansen, NHT, Portelli In Prep.]

- FV momentum $\mathbf{k} = \frac{2\pi\mathbf{n}}{L}$
- Play with action [Davoudi et al. 19]

$$D_{\mu\nu}(k) = \delta_{\mu\nu}\,\frac{1 + w_{|\mathbf{n}|^2}}{k^2}$$

- Why OK? Only care about IV
- $\mathrm{QED_r}$ $w_{|\mathbf{n}|^2} = \delta_{|\mathbf{n}|,1}/6$

$$\overline{c}_0 = \underbrace{c_0}_{=-1} + \sum_{|\mathbf{n}|} w_{|\mathbf{n}|^2} = 0$$



$$Y_3 = \frac{32\pi^2\, m_P\, \overline{c}_0(\mathbf{v}_\ell)}{f_P(1 - r_\ell^4)}\left[F_V^P - F_A^P + 2m_P^2 r_\ell^2\, F_A^{P\,\prime}\right]$$

Figure references

- ⋆ img2go: https://www.img2go.com/ai-art-generator
- ⋆ Fermilab: https://vms.fnal.gov/gallery/view?id=41