# $B^{(*)}$, $D^{(*)}$, $B_s^{(*)}$, $D_s^{(*)}$ Decay Constants and Hyperfine Splittings from Lattice QCD using the Heavy-HISQ Method

**Kerr Miller** Judd Harrison Christine Davies Antonio Smecca

University of Glasgow

HPQCD

DiRAC HPC FACILITY

## Background

The amplitude for the leptonic weak decay of heavy-light meson $H_q$ to final state $F$ can be derived in a low-energy effective theory. The effective weak Hamiltonian, $\mathcal{H}_{\text{eff}}$, can be separated [1] into non-perturbative strong and weak quark interactions, and leptonic electroweak (EW) interactions:

$$\mathcal{A}(H_q \to F) = \langle F|\mathcal{H}_{\text{eff}}|H_q\rangle \approx \langle F|\hat{O}_\ell|0\rangle \cdot \langle 0|\hat{O}_q|H_q\rangle \propto \langle F|\hat{O}_\ell|0\rangle \cdot f_{H_q},$$

at leading order in EW perturbation theory, with $\hat{O}_{\ell,q}$ lepton and quark current operators, and the hadronic matrix element parametrised by $f_{H_q}$, the decay constant (DC) for $H_q$. DCs are defined as the overlap of hadronic states with the vacuum and must be calculated via non-perturbative methods like lattice QCD.

In this work, the $H_q^{(*)}$ is at rest and the lattice axial-vector, vector and tensor current operators, $\{A, V, T\}^{\text{latt}}$, are chosen such that the lattice currents are related to those in the continuum by renormalisation factors $Z_{A,V,T}$ and to the DCs by

$$\langle 0|Z_A\, A^{\text{latt}}|H_q\rangle = \frac{m_h + m_q}{M_{H_q}}\langle 0|P|H_q\rangle = M_{H_q} f_{H_q},$$

$$\langle 0|Z_V\, V^{\text{latt}}|H_q^*\rangle = M_{H_q^*} f_{H_q^*}, \quad \langle 0|Z_T^c\, T^{\text{latt}}|H_q^*\rangle = iM_{H_q^*} f_{H_q^*}^T,$$

where $m_h$ is the heavy quark mass and $q = l, s$ for $H^{(*)}$, $H_s^{(*)}$. $A^{\text{latt}}$ is related to the absolutely normalised pseudoscalar current operator, $P$, by a partially-conserved axial current relation. $Z_T^c$ also accounts for the tensor current's continuum, scheme-dependent renormalisation factor, which is usually given in the $\overline{\text{MS}}$ scheme. The renormalisation factors $Z_V$, $Z_T^c$ were determined previously [2, 3].

Precise determinations of the tensor-to-vector DC ratios will allow us to constrain new physics effects on effective Wilson coefficients. For example, the coefficients $C_{7,9}^{\text{eff}}$ appear in the total width for the rare decay $B_s^* \to \ell^+\ell^-$ and can be better constrained with knowledge of the ratio $f_{B_s^*}^T/f_{B_s^*}$. The ratios can also be combined with precise determinations of the pseudoscalar DCs to yield high-precision values of the vector and tensor DCs.

Additionally, we calculate the 'hyperfine splittings' (HFSs), the mass differences between a pseudoscalar and its associated vector:

$$\Delta_{H_q^*-H_q} = M_{H_q^*} - M_{H_q}.$$

These provide further precision tests of the Standard Model and important inputs to theoretical calculations (e.g., in Heavy Quark Effective Theory).

## Correlator Fits

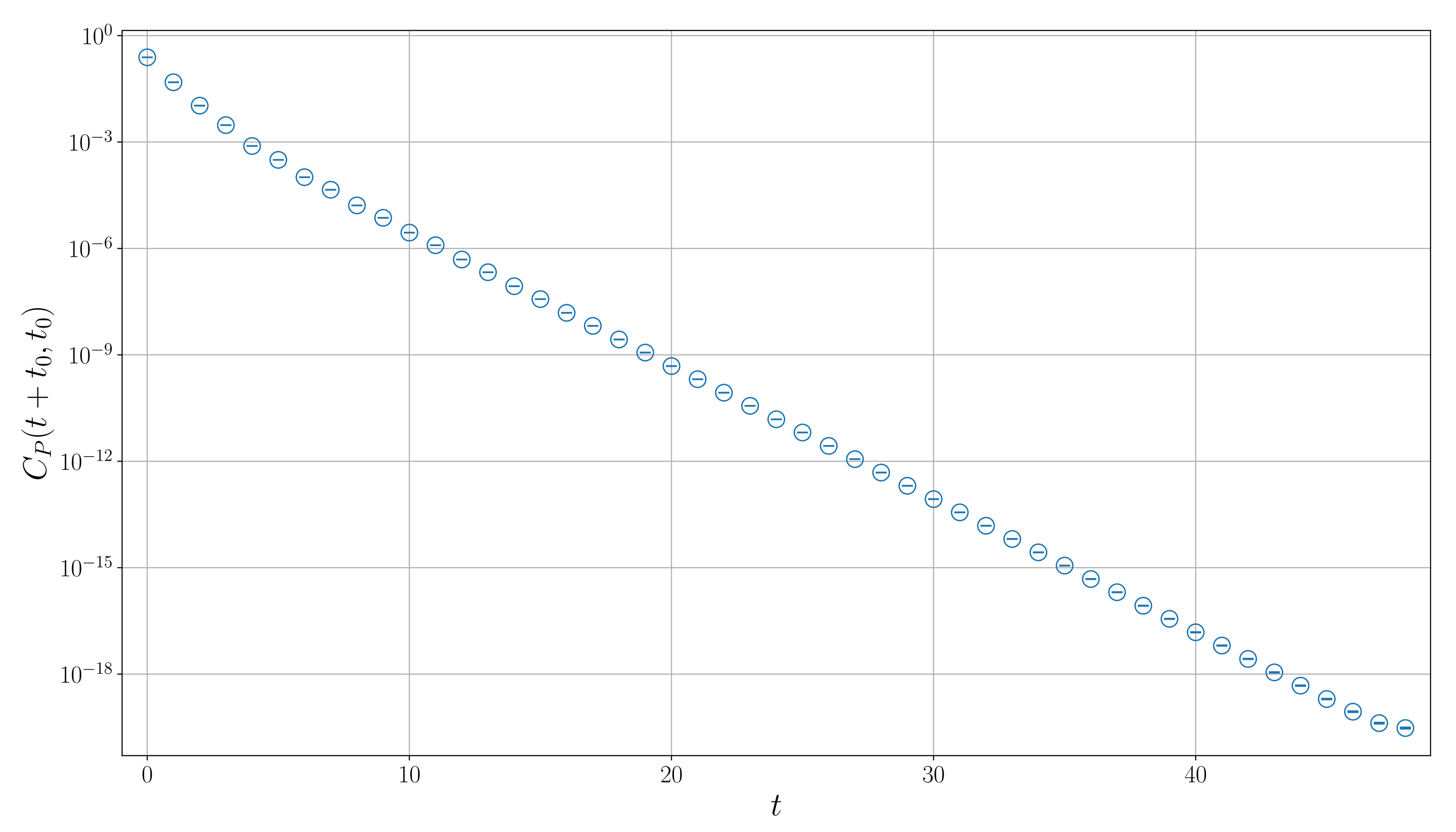


Figure 1: A log plot of raw correlator data for the $D$ meson on one of our ensembles. This is the correlator over one half of the lattice; the other half is its mirror image, due to periodic boundary conditions. The errors are barely visible on this scale.

We first calculate $H^{(*)}$ and $H_s^{(*)}$ 2-point correlation functions on the lattice. These are then fitted to the form

$$C_J(t+t_0, t_0) = \langle \mathcal{O}_J(t+t_0)\,\mathcal{O}_J^\dagger(t_0)\rangle = \sum_n \left[\left|A_n^J\right|^2 \exp\left(-tM_n^J\right) - (-1)^t \left|\tilde{A}_n^J\right|^2 \exp\left(-t\tilde{M}_n^J\right)\right],$$

where $\mathcal{O}_J(x) = \bar{q}\,\Gamma^J h(x)$ is a local meson interpolating operator; $\Gamma^J = \gamma^5, \gamma^i, \gamma^i\gamma^t$ is the Dirac structure corresponding to the pseudoscalar, vector or tensor current, respectively; $n = 0, 1, 2, \ldots$ is the excitation label ($n = 0$ is the ground state); and $A_n^J$, $M_n^J$ denote an amplitude and a mass for given $J$, $n$. The $\tilde{A}$, $\tilde{M}$ quantities are those of additional, time-doubled states that couple to the interpolating operators when using staggered quark currents. The ground state masses, $M_0^J$, and amplitudes, $A_0^J$, that we extract from the correlator fits are related to the decay constants:

$$\left|A_0^J\right|^2 \exp\left(-tM_0^J\right) = \frac{\left|\left\langle 0\left|\bar{q}\,\Gamma h\right|H_q^{(*)}\right\rangle\right|^2}{2M_{H_q^{(*)}}} \exp\left(-tM_{H_q^{(*)}}\right)$$

$$\implies A_0^P = \frac{M_{H_q}^{3/2}}{\sqrt{2}\,(m_h + m_q)} f_{H_q}, \qquad A_0^V = \frac{\sqrt{M_{H_q^*}}}{\sqrt{2}\,Z_V} f_{H_q^*}, \qquad A_0^T = \frac{\sqrt{M_{H_q^*}}}{\sqrt{2}\,Z_T^c} f_{H_q^*}^T.$$

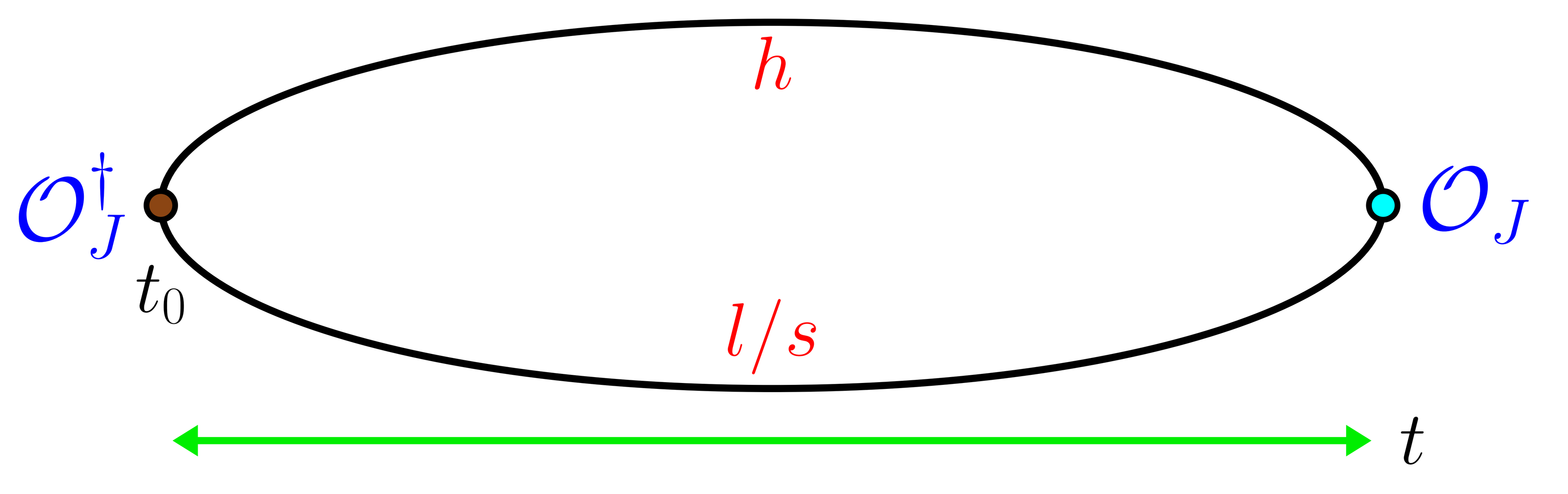


Figure 2: Schematic 2-point correlator constructed to calculate DCs. Meson created by operator $\mathcal{O}_J^\dagger$ at time $t_0$ (brown circle, left). Black lines represent quark propagators for heavy and light/strange quarks. Meson annihilated by operator $\mathcal{O}_J$ at time $t_0 + t$ (cyan circle, right).

## The Heavy-HISQ Method

Once the DC ratios and HFSs are calculated from the extracted masses and amplitudes, they are fitted to a form that is inspired by Heavy Quark Effective Theory and accounts for quark mass mistunings, discretisation effects, and chiral analytic dependence. We then take the limit as $a \to 0$, where $a$ is the lattice spacing, to obtain results for the DC ratios and HFSs in the continuum.

We use the **H**ighly **I**mproved **S**taggered **Q**uark (HISQ) action, which provides important advantages over other lattice quark formalisms:

- HISQ has very small discretisation effects, can reach the physical $b$ quark mass.
- HISQ is more numerically efficient, enabling high statistics with both unphysical and physical light (degenerate up/down) quarks.
- All-HISQ currents permit fully non-perturbative renormalisation, avoiding the dominant systematic uncertainty of non-relativistic methods.

Our 'heavy-HISQ' method employs a range of heavy quark masses, from the physical charm to the physical bottom: $m_c \leq m_h \leq m_b$. This allows us to map out the heavy-mass dependence of the DC ratios and HFSs, and provides excellent control over systematic uncertainties at the physical extremes of the mass range.

Until now, lattice computations at the physical $b$ quark mass would have been prohibitively expensive to simulate. However, recent work by members of HPQCD has shown that the physical $b$ mass may be reached while maintaining modest discretisation effects on the finest lattices we use.

## (PRELIMINARY) Decay Constant Ratios and Hyperfine Splittings

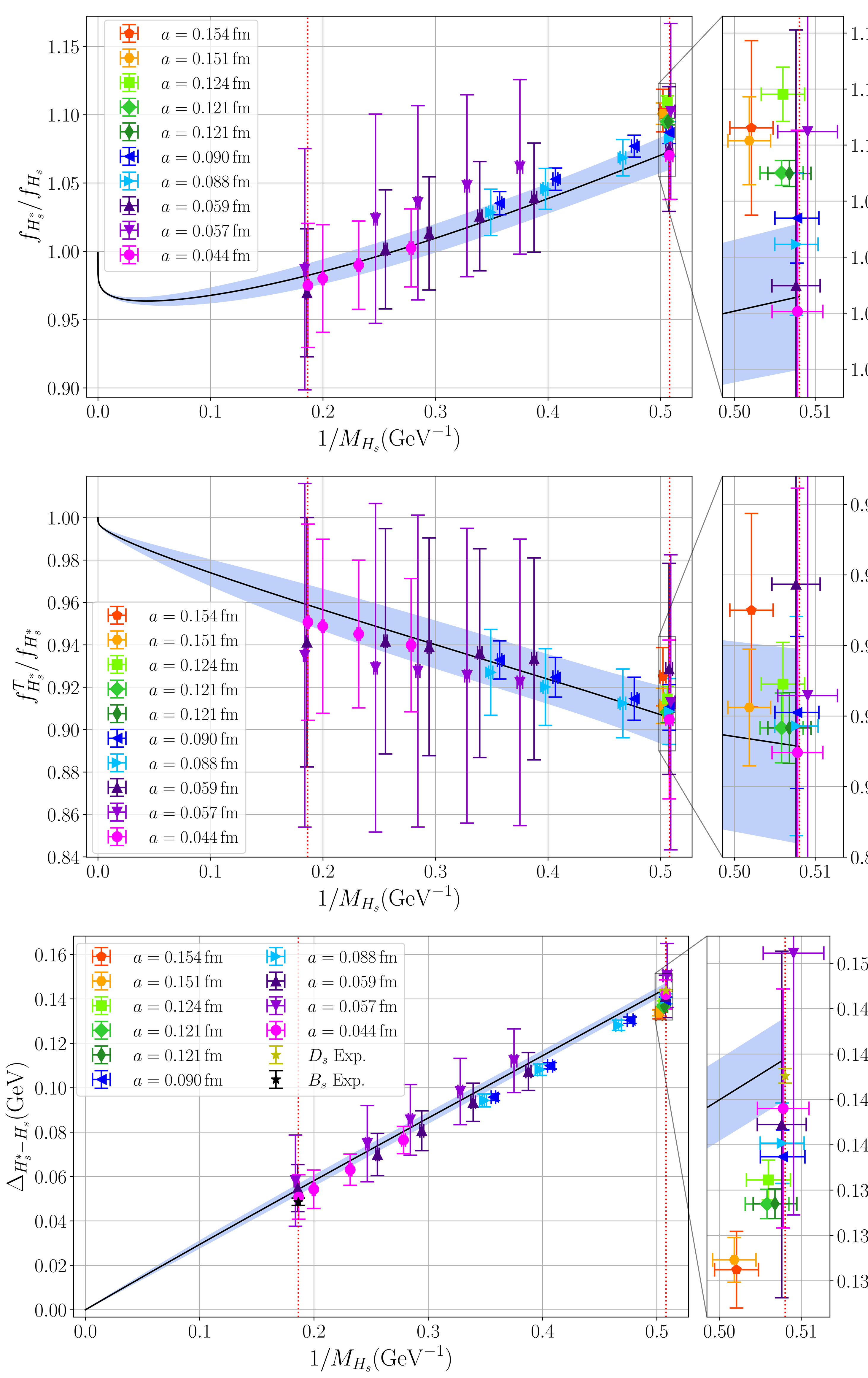


Figure 3: The vector-to-pseudoscalar DC ratio (top), tensor-to-vector DC ratio (middle) and HFS (bottom) for heavy-strange mesons. We have lattice data at multiple heavy-quark masses, with the blue bands showing the results extrapolated to the continuum ($a \to 0$). The red dotted lines indicate $1/M_{B_s}$ (left) and $1/M_{D_s}$ (right), the legend shows the approximate lattice spacings to which the data correspond and the panels on the right of each plot show a magnified view of the plot at $1/M_{H_s} = 1/M_{D_s}$. Experimental data points are included on the HFS plot, given by the black and gold stars.

## Conclusions

- We have computed decay constant ratios and hyperfine splittings for $B^{(*)}$, $D^{(*)}$, $B_s^{(*)}$ and $D_s^{(*)}$ mesons, providing insight into Standard Model flavour phenomenology, sensitivity to new physics and valuable inputs for future calculations.
- Our preliminary extrapolations are well-controlled across the physical mass range.
- To our knowledge, this is the first lattice calculation enabling a determination of the $B^*$ and $B_s^*$ tensor decay constants in the Standard Model — and with high-precision!

[1] T. Blake, G. Lanfranchi, and D. M. Straub, "Rare $B$ Decays as Tests of the Standard Model," *Prog. Part. Nucl. Phys.*, vol. 92, pp. 50–91, 2017.

[2] D. Hatton, C. T. H. Davies, G. P. Lepage, and A. T. Lytle, "Renormalizing vector currents in lattice QCD using momentum-subtraction schemes," *Phys. Rev. D*, vol. 100, no. 11, p. 114513, 2019.

[3] D. Hatton, C. T. H. Davies, G. P. Lepage, and A. T. Lytle, "Renormalization of the tensor current in lattice QCD and the $J/\psi$ tensor decay constant," *Phys. Rev. D*, vol. 102, no. 9, p. 094509, 2020.

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