Lattice Gauge Theories beyond the Standard Model

Biagio Lucini
TELOS (τέλος) Collaboration

DiRAC Day, Cardiff (UK), 12th December 2024

The TELOS collaboration

Theoretical Explorations on the Lattice with Orthogonal and Symplectic Groups

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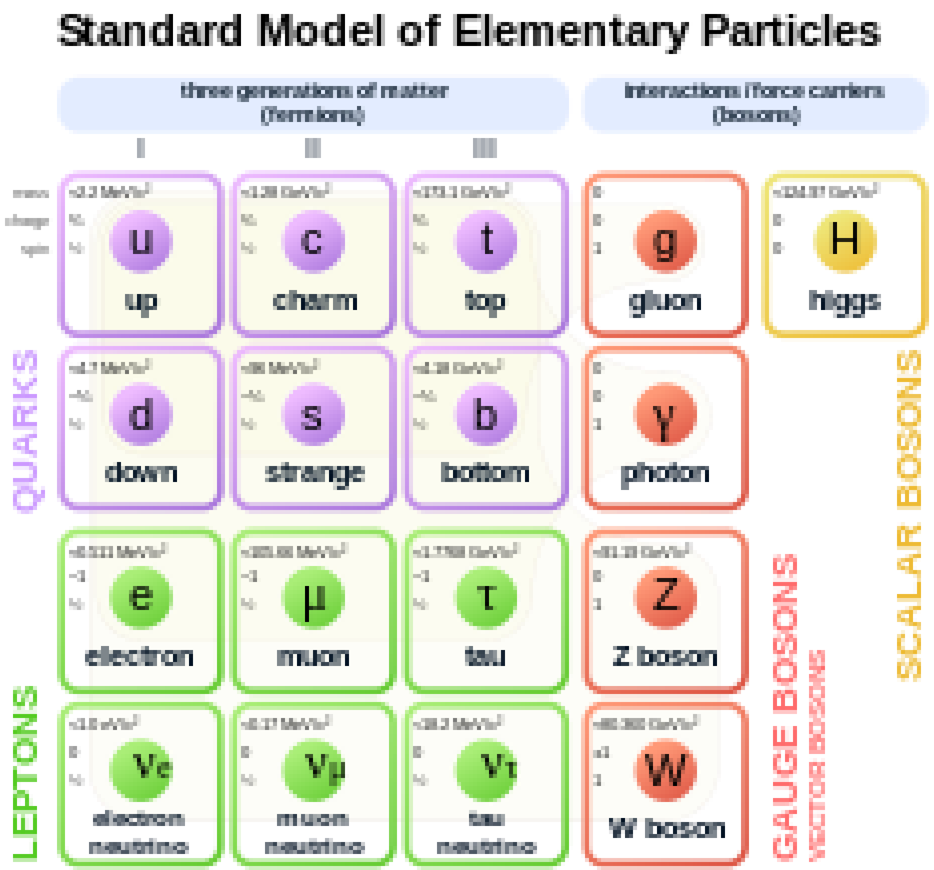
Review: E. Bennett *et al.,* Universe 8 (2023) [arXiv:2304.01070]

The Standard Model of Particle Physics



- The SM describes the strong and electroweak interactions (including the Higgs sector) very successfully
- The electroweak interactions are amenable to analytical predictions
- The strong interactions require non-perturbative ab initio methods

The Lattice as a computational paradigm

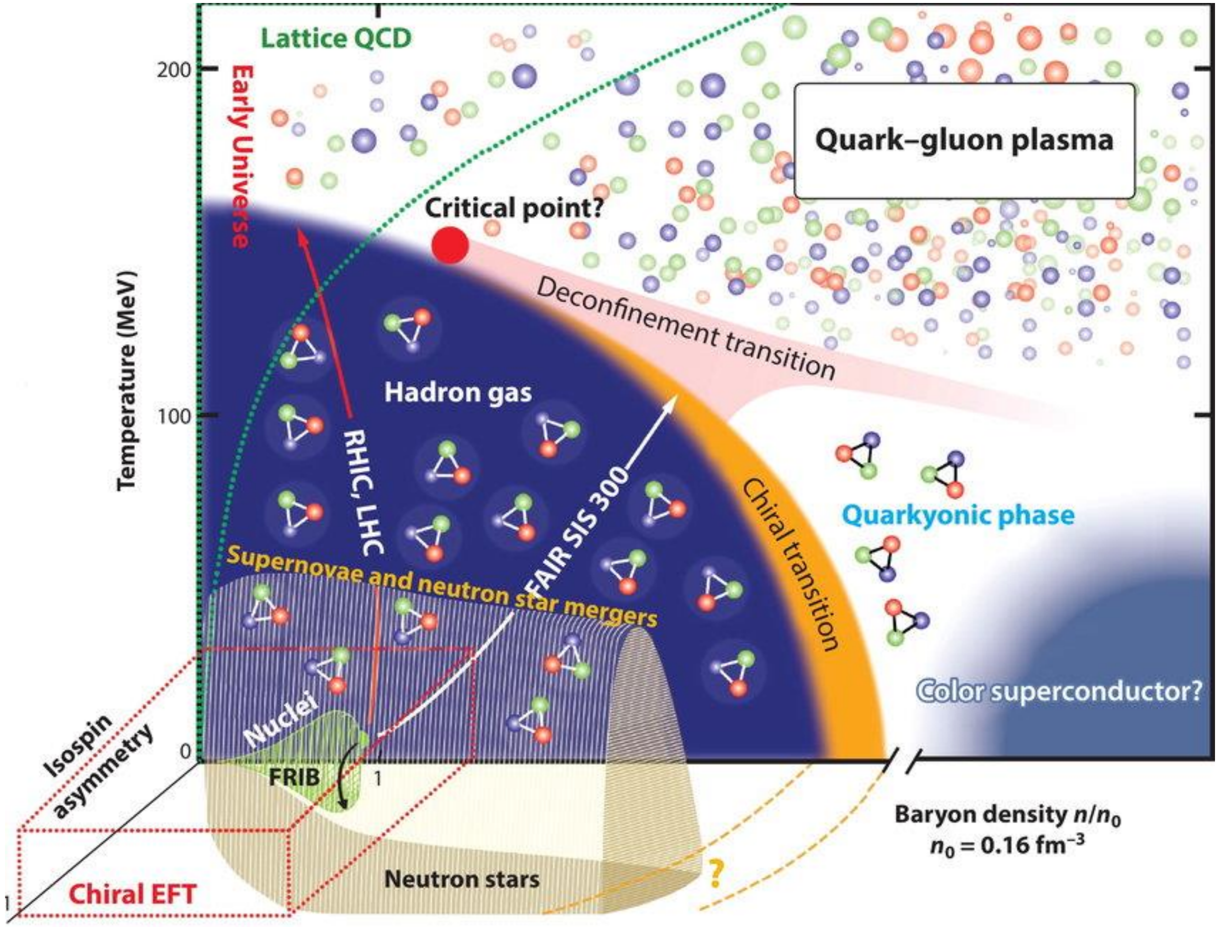


Image credits: Report of the Topical Group on Cosmic Probes of Fundamental Physics (Snowmass 2021)

- The theory gives rise to a spectrum of hadrons (baryons, mesons, glueballs and exotica) at zero temperature
- Deconfinement happens at a critical/crossover temperature
- Exotic phases at non-zero density
- All these are non-perturbative phenomena

Enters the Lattice…

Open problems in the Standard Model

- Gravity is not accounted for
- Asymmetry matter-antimatter
- Absence of CP violation in QCD
- Dark matter/dark energy
- Fundamental mechanism for electroweak symmetry breaking
- …

Investigation paradigms

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1. Precision frontier

Observables are computed theoretically and determined experimentally to very high accuracy, with deviations providing evidence for new physics and agreement setting stringent bounds for the latter

The Lattice provides a robust calculation tool for precision physics

2. Energy frontier

Theoretically motivated interactions beyond the standard model are studied, with their observables providing input to phenomenology

Non-perturbative calculation on a lattice very often crucial

This talk: Higgs compositeness from beyond the standard model strongly interacting theories

The Higgs as a pseudo-Nambu-Goldstone boson

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Little Hierarchy Problem: If the Higgs boson is composite, its mass must be significantly lower than that of other states of the novel strong interaction

Possible solution: the Higgs is a χSB PNGB arising in a new strong interaction

Patterns of χSB $\mathcal{G} \mapsto \mathcal{H}$ for a theory with N_f Dirac fermions

1. SU(N) gauge group: $\mathrm{SU}(N_f)_V \times \mathrm{SU}(N_f)_A \mapsto \mathrm{SU}(N_f)_V$
2. Real gauge group: $\mathrm{SU}(2\,N_f) \mapsto \mathrm{SO}(2\,N_f)$
3. Pseudoreal gauge group: $\mathrm{SU}(2\,N_f) \mapsto \mathrm{Sp}(2\,N_f)$

Embedding of the standard model: $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \subset \mathcal{H} \subset \mathcal{G}$
$\hookrightarrow$ The physical Higgs is identified with four of the pions

In this talk we discuss an Sp(4) gauge theory with $N_f = 2$ fundamental fermions
$\hookrightarrow$ Expected χSB pattern: $\mathrm{SU}(4) \mapsto \mathrm{Sp}(4)$

Extra PNGBs or glueballs are dark matter candidates

Top partial compositeness

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- Question: why is the mass of the top quark at the electroweak scale?
- Potential answer: the mass of the top is lifted through some interaction with beyond the standard model top partners
- Top partner can take the form of composite baryons with fermions in multiple representations (needs large anomalous scaling)

Viable models

Coset	HC	ψ	χ	$-q_\chi/q_\psi$	Baryon	Name
$\frac{SU(5)}{SO(5)} \times \frac{SU(6)}{SO(6)}$	SO(7)	$5 \times \mathbf{F}$	$6 \times \mathbf{Sp}$	5/6	$\psi\chi\chi$	M1
	SO(9)			5/12		M2
	SO(7)	$5 \times \mathbf{Sp}$	$6 \times \mathrm{F}$	5/6	$\psi\psi\chi$	M3
	SO(9)			5/3		M4
$\frac{SU(5)}{SO(5)} \times \frac{SU(6)}{Sp(6)}$	Sp(4)	$5 \times \mathbf{A}_2$	$6 \times \mathbf{F}$	5/3	$\psi\chi\chi$	M5
$\frac{SU(5)}{SO(5)} \times \frac{SU(3)^2}{SU(3)}$	SU(4)	$5 \times \mathbf{A}_2$	$3 \times (\mathbf{F}, \overline{\mathbf{F}})$	5/3	$\psi\chi\chi$	M6
	SO(10)	$5 \times \mathbf{F}$	$3 \times (\mathbf{Sp}, \overline{\mathbf{Sp}})$	5/12		M7
$\frac{SU(4)}{Sp(4)} \times \frac{SU(6)}{SO(6)}$	Sp(4)	$4 \times \mathbf{F}$	$6 \times \mathbf{A}_2$	1/3	$\psi\psi\chi$	M8
	SO(11)	$4 \times \mathbf{Sp}$	$6 \times \mathbf{F}$	8/3		M9
$\frac{SU(4)^2}{SU(4)} \times \frac{SU(6)}{SO(6)}$	SO(10)	$4 \times (\mathbf{Sp}, \overline{\mathbf{Sp}})$	$6 \times \mathbf{F}$	8/3	$\psi\psi\chi$	M10
	SU(4)	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$6 \times \mathbf{A}_2$	2/3		M11
$\frac{SU(4)^2}{SU(4)} \times \frac{SU(3)^2}{SU(3)}$	SU(5)	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$3 \times (\mathbf{A}_2, \overline{\mathbf{A}_2})$	4/9	$\psi\psi\chi$	M12

G. Ferretti, T. Karataev, arXiv:1312.5330

G. Cacciapaglia, G. Ferretti, T. Flake and H. Serodio, arXiv:1902.06890

Sp(2N) groups

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Sp$(2N)$ can be defined as the subgroup of SU$(2N)$ whose elements U fulfil the condition

$$U\Omega U^T = \Omega\,, \qquad \Omega = \begin{pmatrix} 0 & \mathbb{I} \\ -\mathbb{I} & 0 \end{pmatrix}$$

This constrains the structure of U as follows:

$$U = \begin{pmatrix} A & B \\ -B^* & A^* \end{pmatrix}\,, \qquad \text{with } AA^\dagger + BB^\dagger = \mathbb{I} \text{ and } A^TB = B^TA$$

For an element **u** in the algebra, this implies

$$\mathbf{u} = \begin{pmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{b}^* & -\mathbf{a}^* \end{pmatrix}\,, \qquad \text{with } \mathbf{a} = \mathbf{a}^\dagger \text{ and } \mathbf{b} = \mathbf{b}^T$$

Other properties:

1. The dimension of the group is $N(2N+1)$
2. The group is pseudoreal
3. The group has rank N $\Rightarrow$ N independent SU(2) subgroups

From the continuum to the lattice (and back)

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1. Start from the Euclidean Path Integral formulation of the theory

$$\langle O \rangle = \frac{\int (\mathcal{D}\phi)\, O[\phi] e^{-S}}{\int (\mathcal{D}\phi)\, e^{-S}}$$

2. Approximate the integral on a grid of spacing a and of size $V = N_t \times N_s^3$

3. Compute the integral with Monte Carlo methods

4. Extrapolate to $V \to \infty$ and $a \to 0$

Fields on the lattice

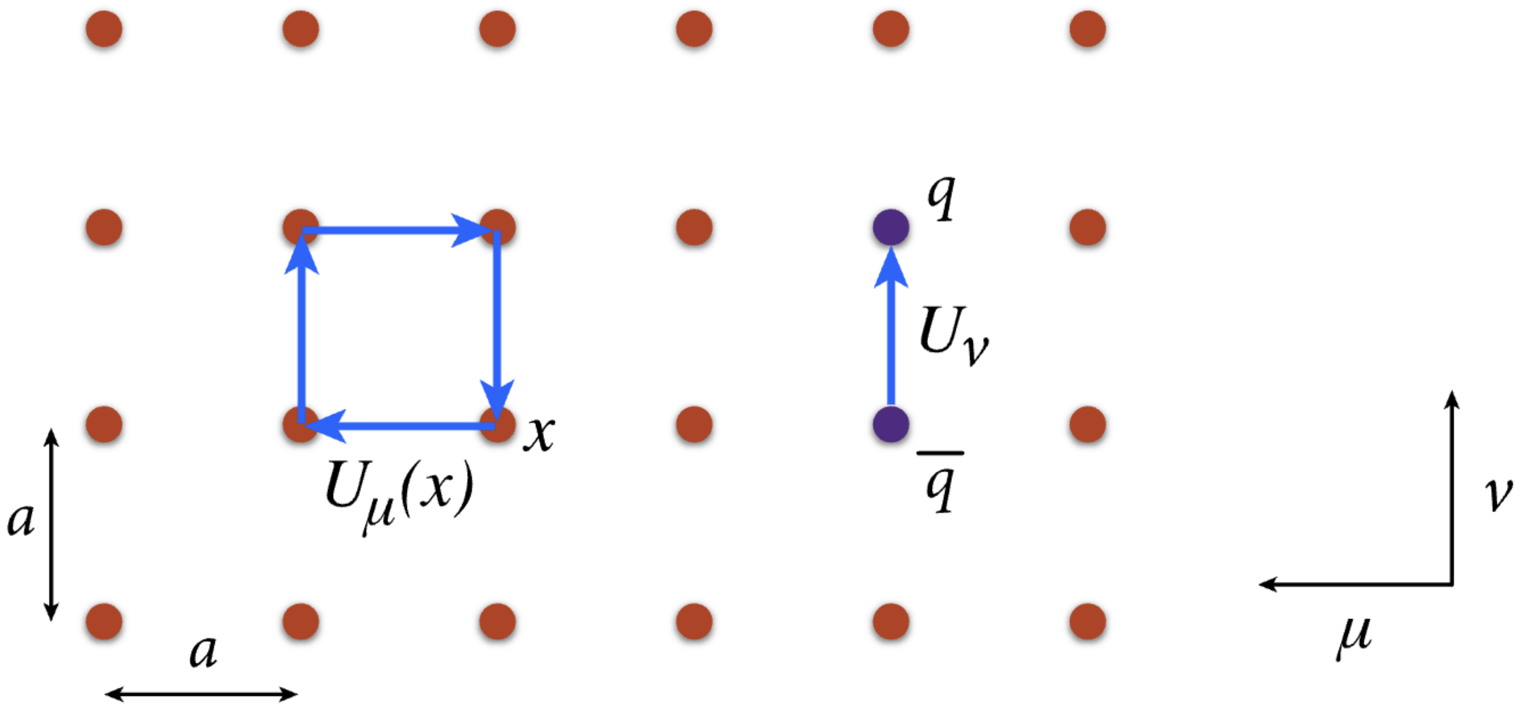

Figure from Particle Data Group

Action of the Model

$$S = \beta \sum_{x} \sum_{\mu<\nu} \left(1 - \frac{1}{4} \mathrm{Re}\,\mathrm{Tr}\,\mathcal{P}_{\mu\nu} \right)$$

$$+ \sum_{j=1}^{n_f} \sum_{x} \overline{\Psi^j}(x) D_m^{(as)} \Psi^j(x)$$

$$+ \sum_{i=1}^{N_f} \sum_{x} \overline{Q^i}(x) D_m^{(f)} Q^i(x)$$

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Pure gauge contribution

Action of the Model

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Fundamental fermion part

Action of the Model

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$$S = \beta \sum_{x} \sum_{\mu<\nu} \left(1 - \frac{1}{4} \mathrm{Re}\,\mathrm{Tr}\,\mathcal{P}_{\mu\nu} \right)$$

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$$+ \sum_{i=1}^{N_f} \sum_{x} \overline{Q^i}(x) D_m^{(f)} Q^i(x)$$

Antisymmetric fermion part

Action of the Model

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$$S = \beta \sum_{x} \sum_{\mu<\nu} \left(1 - \frac{1}{4} \mathrm{Re}\,\mathrm{Tr}\,\mathcal{P}_{\mu\nu}\right)$$

$$+ \sum_{j=1}^{n_f} \sum_{x} \overline{\Psi^j}(x) D_m^{(as)} \Psi^j(x)$$

$$+ \sum_{i=1}^{N_f} \sum_{x} \overline{Q^i}(x) D_m^{(f)} Q^i(x)$$

→ Combined fundamental and antisymmetric contribution

Example: masses from correlators

$$\langle \phi^\dagger(0)\phi(x)\rangle = \sum_n |c_n|^2 e^{-m_n x}$$

$$\overset{x\to\infty}{=} |c_0^2| e^{-m_0 x}$$

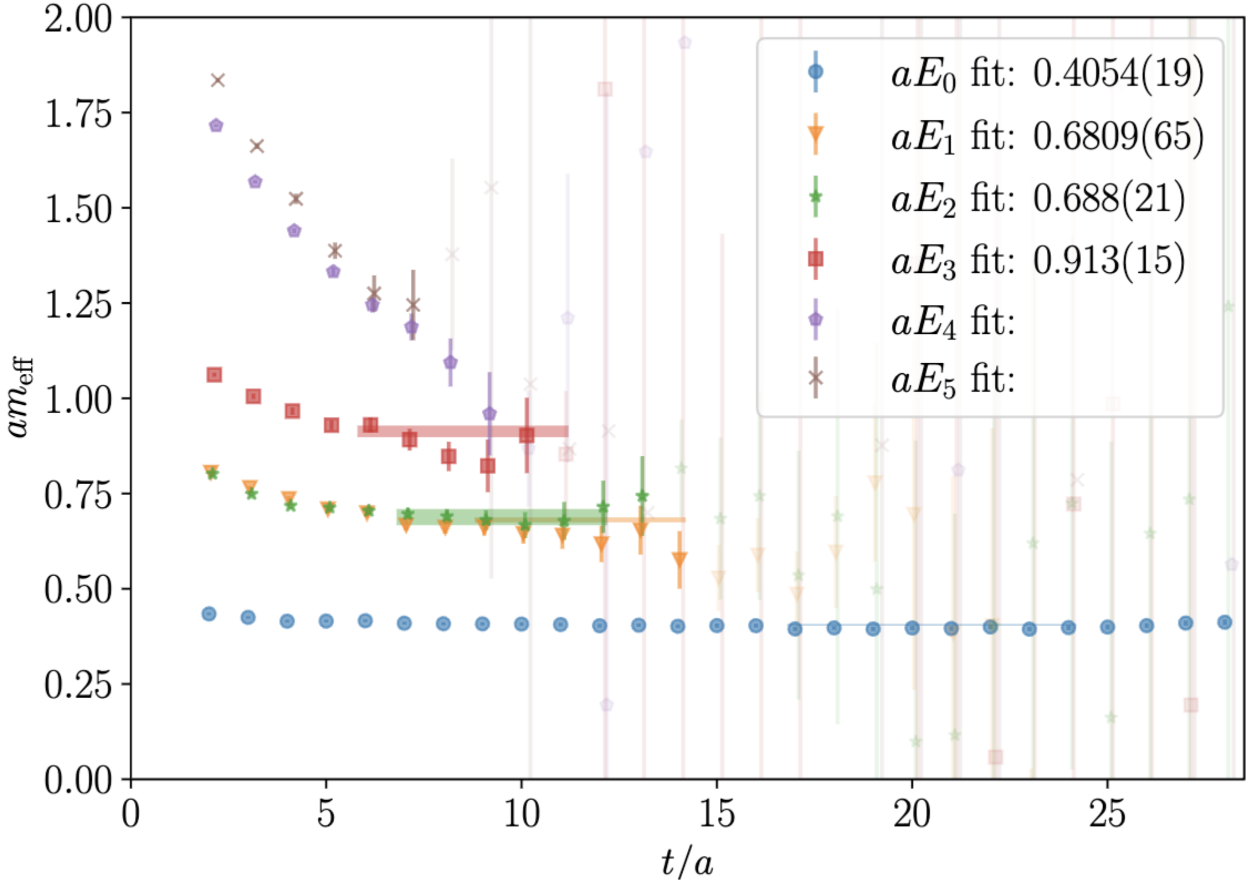

Figure from E. Bennett et al., Phys. Rev. D 110 (2024) 7, 074504 (arXiv:2405.01388)

Spectrum of the model

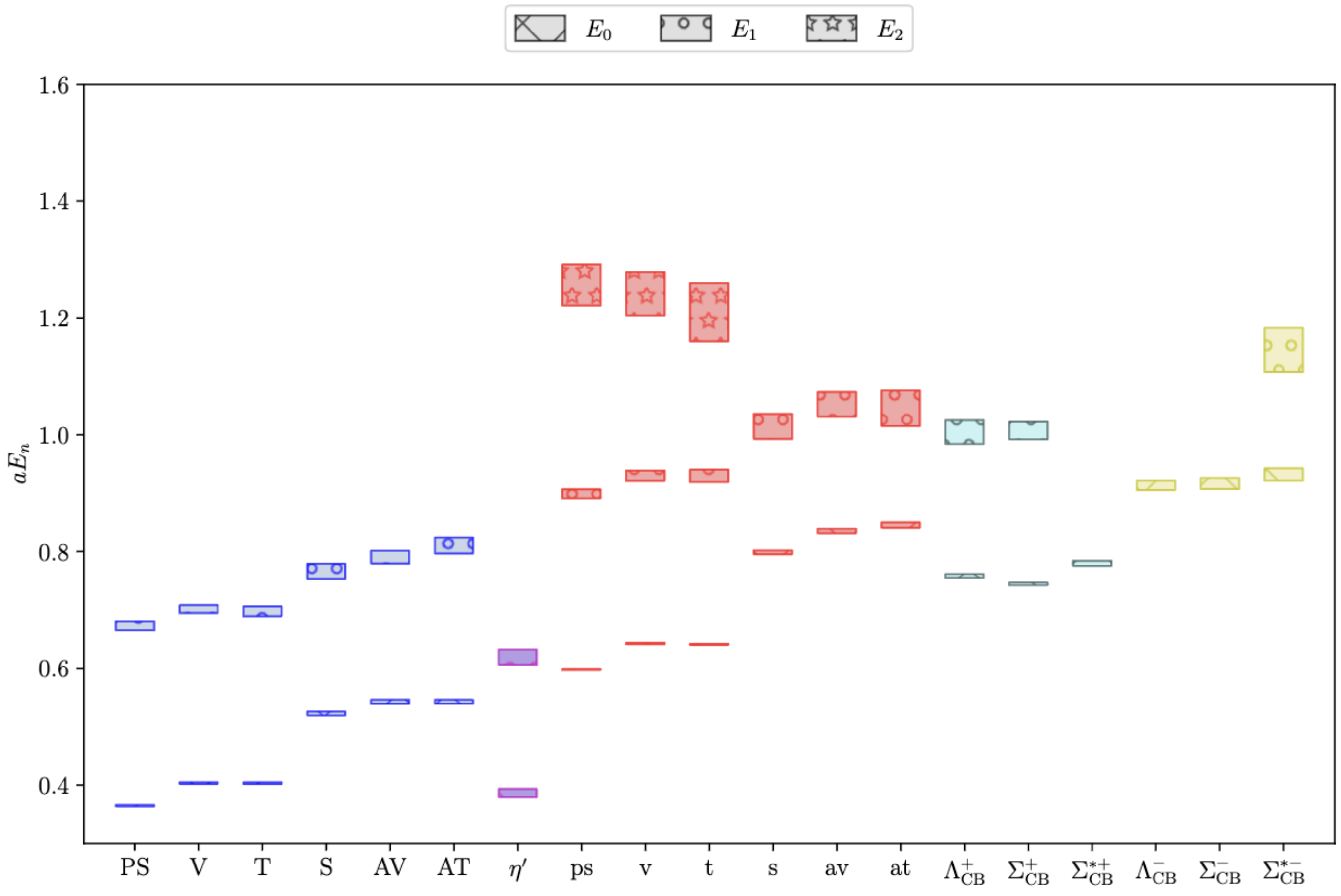

Figure from H. Hsiao *et al.*, arXiv:2411.18379

Glueballs in Sp(2N) Yang-Mills

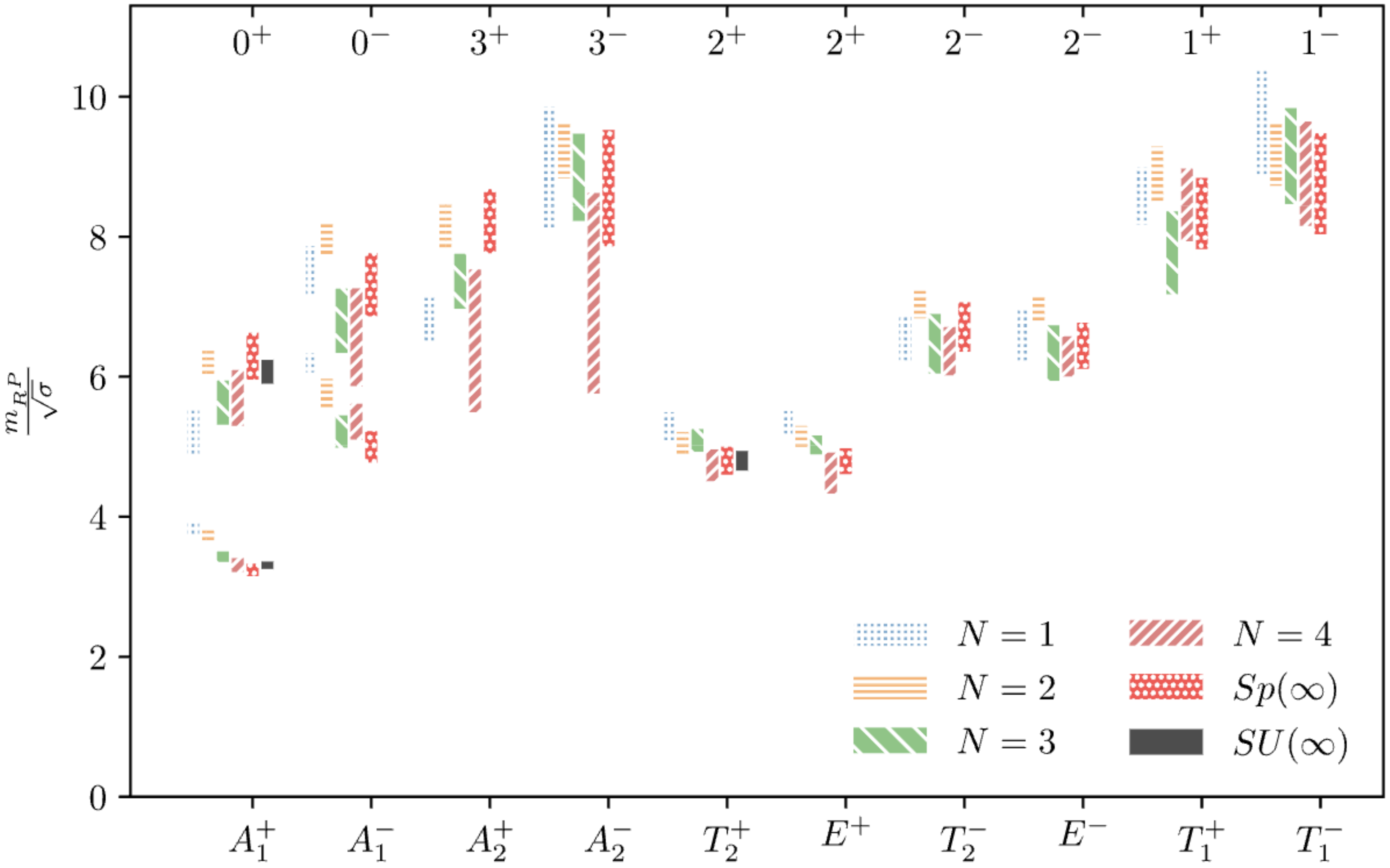

Figure from E. Bennett *et al*., Phys.Rev.D 102 (2020) 1, 011501 [arXiv:2004.11063]

Summary

- Strongly interacting dynamics beyond the standard model provides viable mechanisms of Higgs compositeness
- A programme is being undertaken on DiRAC to characterise a template model of Higgs compositeness based on the pseudo-Nambu-Goldstone mechanism
- Advanced techniques have been designed and developed to extract the spectrum of the model
- Next steps:
  - Lowering the mass of the fermions
  - Study larger lattices
  - Approach the continuum limit in a controlled way

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