SU(2) with adjoint Domain Wall Fermions

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Context

SU(2) with adjoint Dirac matter is a family of theories with a number of interesting aspects:

- It can be used as the basis for beyond the standard model theories of electroweak symmetry breaking
- It can help to pin down the lower end of the conformal window in SU(2)
- $N_{\mathrm{f}} = 1$ may be dual to the critical theory of a topological phase transition
- $N_{\mathrm{f}} = 2$ is likely in the conformal window
- Previous work focuses on Wilson fermions, which struggle near the chiral limit [1, 2]
- This is reached the limit of what can be studied with Wilson fermions with reasonable resources

Lattice theory

We simulate using the standard Wilson gauge action

$$S_{\mathrm{G}} = \beta \sum_{p} \mathrm{Tr}\left[1 - \frac{1}{2}U(p)\right]$$

and the Möbius Domain Wall Fermion action

$$S_{\mathrm{F}} = \sum_{x,y} \overline{\psi}(x) D_{\text{Möbius}}(x,y)\psi(y)$$

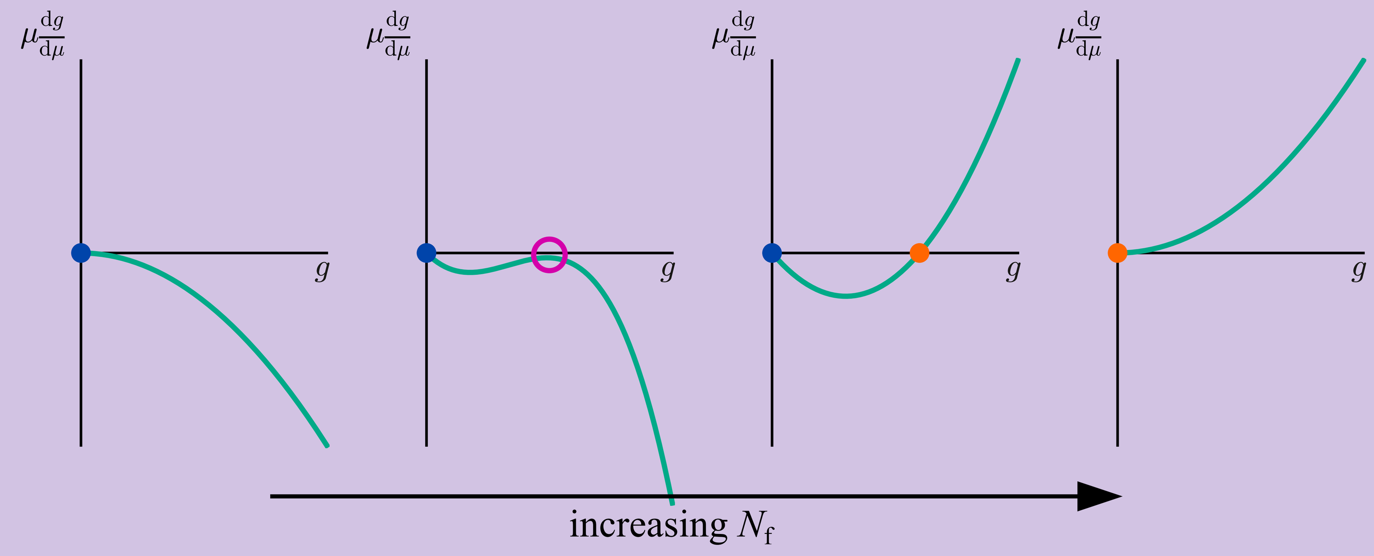
$$D_{\text{Möbius}} = \frac{(b+c)D_{\mathrm{W}}(M_5)}{2+(b-c)D_{\mathrm{W}}(M_5)}$$

where D_{W} is the Wilson fermion kernel

$$D_{\mathrm{W}}(M_5) = (4+M_5)\delta_{x,y} - \frac{1}{2}\Big[(1-\gamma_{\mu(x)}\delta_{x+\mu,y}) + (1+\gamma_\mu)U_\mu^\dagger \delta_{x,y+\mu}\Big]$$

This should give better chiral behaviour than previous work with Wilson fermions, which have limited our approach to zero fermion mass (where conformal behaviour is most visible)

The conformal window

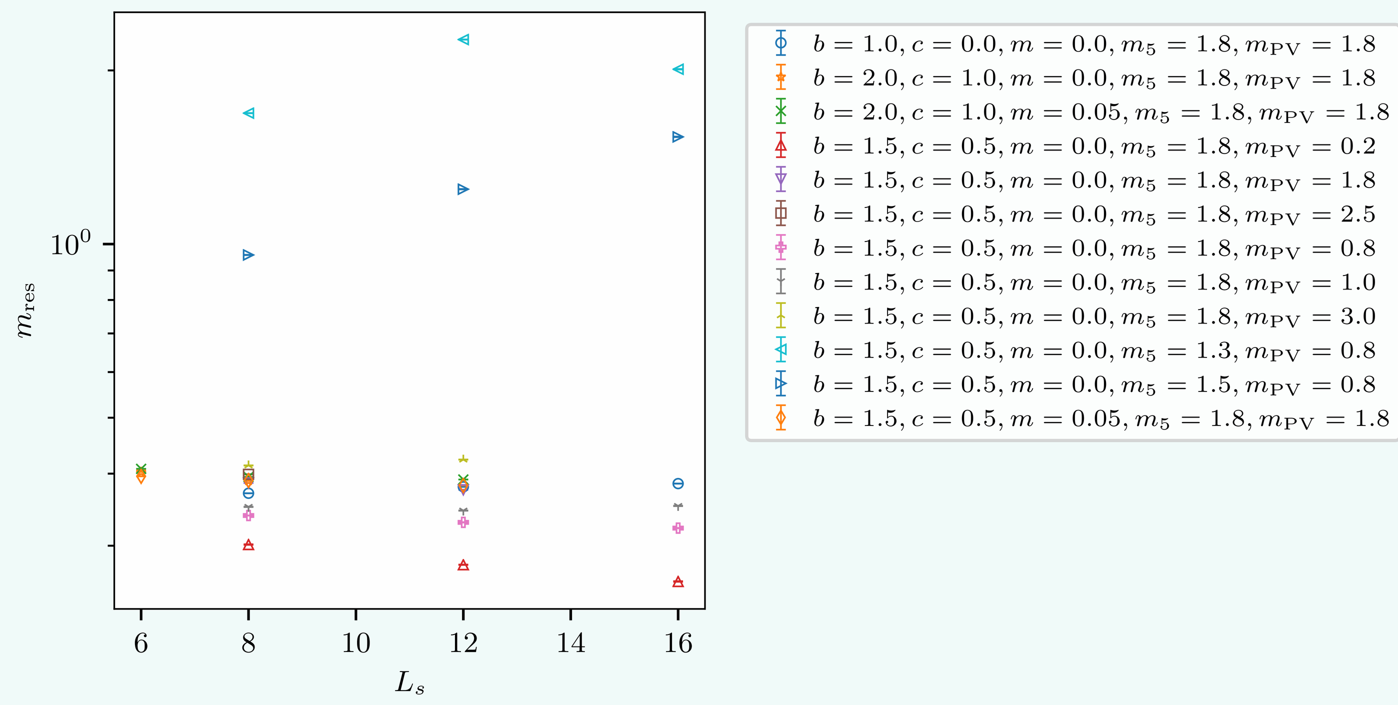


While in QCD the beta function is always non-positive, higher values of N_{f} show a region where there is a conformal fixed point. Immediately below this conformal window there may be a region where there is near-conformal behaviour, a large mass anomalous dimension, and slow-running ("walking") coupling.

Implementation

- In $N_{\mathrm{f}} = 1$ we use the Exact One-Flavour Algorithm and one additional Pauli–Villars mass
- In $N_{\mathrm{f}} = 2$ we use the Hybrid Monte Carlo algorithm
- Implemented using Grid [3]
- Performed on the DiRAC Extreme Scaling Service Tursa using A100 GPUs

Tuning parameters for $N_{\mathrm{f}} = 1$



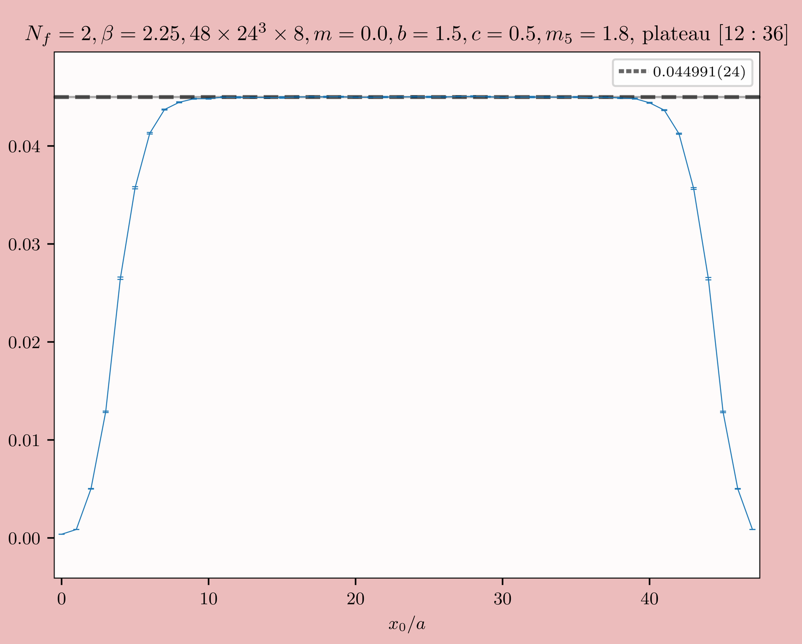
An initial parameter scan shows worse-than-expected scaling of m_{res} with the domain wall separation for all parameters considered. Currently m_{res} is too high to move to large-volume production workloads. The scan is in the process of being extended to larger values of L_5 and higher values of b, c in search of suitably low residual masses.

Residual Masses

Even with Möbius Domain Wall Fermions, approaching the zero fermion mass limit is not automatic. We must tune the algorithmic parameters b, c, and M_5, as well as the Pauli-Villars mass M_{PV} and the domain wall separation L_5, to minimize the residual chiral symmetry breaking effects. This residual is measured as

$$m_{\mathrm{res}}(t) = \frac{\sum_{\vec{x}} \left\langle j_5(\vec{x}, t, L_5/2) j_5(\vec{0}, t, L_5)\right\rangle}{\sum_{\vec{x}} \left\langle \overline{q}_{\vec{x},t}\gamma_5 q_{\vec{x},t}\overline{q}_0\gamma_5 q_0\right\rangle}$$

in a plateau region, e.g.



Residual masses are implemented using Hadrons [4]. We scan a region of parameter space for each theory in search of a sufficiently low residual mass to enable simulations with physically meaningful results.

References

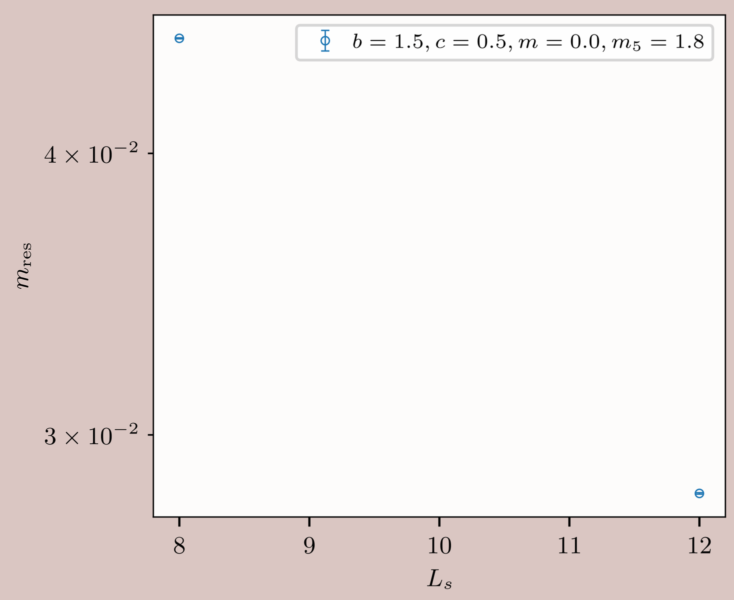
[1] AA, EB, GB, BL, Phys.Rev.D 104 (2021) 7, 074519

[2] AA, EB, GB PB, BL, In preparation

[3] Grid: https://github.com/paboyle/Grid

[4] Hadrons: https://github.com/aportelli/Hadrons

Tuning parameters for $N_{\mathrm{f}} = 2$



Very preliminary results show more promising scaling than the $N_{\mathrm{f}} = 1$ case; this forms the basis of a case for a work package in a followup DiRAC project.

Conclusions

- In $N_{\mathrm{f}} = 2$ we see good scaling of m_{res}
  - Near ready to start production runs
- In $N_{\mathrm{f}} = 1$ the scaling is less well understood
  - Further parameter studies are needed to understand optimal parameters for production

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