$Sp(2N)$ Glueballs

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The discovery of the Higgs boson in 2012 provided evidence for our understanding of the origin of mass. Despite the Standard Model's tremendous success, there still remain some unanswered questions one of which is the 'Naturalness Problem'.

Motivation

The Naturalness problem

One of the motives for studying the Symplectic groups is an explanation for the Higgs boson's low mass of 125 GeV.

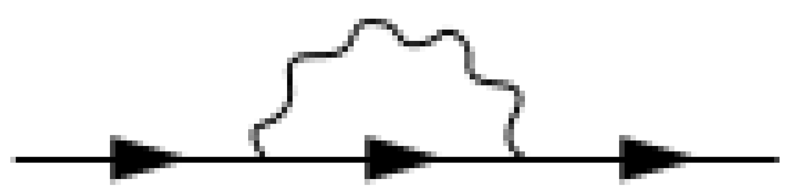

Figure 1: *Electron self energy [1].*

Figure 1 is the familiar Feynman diagram for an electron interacting with its own electromagnetic field. This phenomenon gives us cause to redefine the electron's mass and electric charge. After renormalisation, we arrive at the observable mass/charge as opposed to the bare quantities. The same process must be carried out for the Higgs boson. The issue that arises here is how many corrections to the Higgs mass we must include.

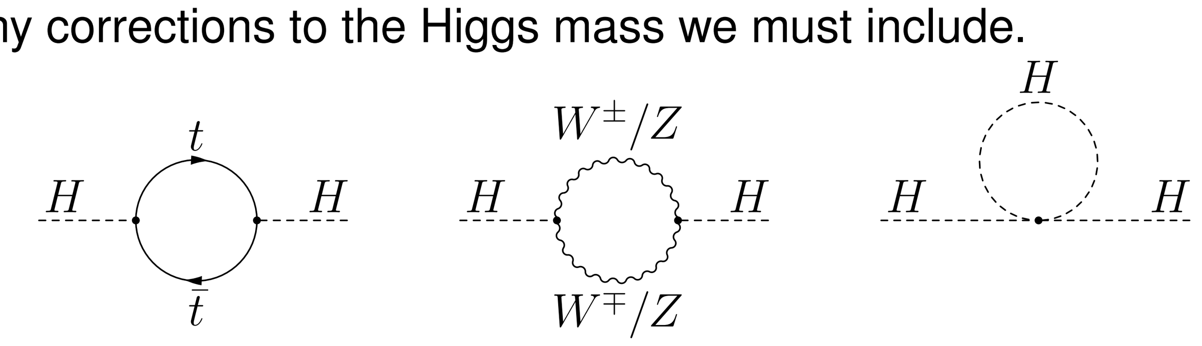

Figure 2: *Example corrections to the Higgs boson's mass.*

Figure 2 shows loop corrections to the mass of the Higgs boson. Left to right: the fermion loop (which is dominated by the top quark due to its very large Yukawa coupling), the loop of weak gauge bosons and the Higgs quartic self interaction.

Each of the above diagrams in figure 2 requires renormalisation just as figure 1 does. The three diagrams diverge quadratically which requires a quadratically rising counter-term to yield a finite result.
The correction to the Higgs mass (squared) due to the diagrams in figure 2 takes the form [2]

$$\delta m_H^2 = \frac{3\Lambda_{\text{SM}}^2}{8\pi^2}\left[y_t^2 - g_W^2\left(\frac{1}{4}+\frac{1}{8\cos^2\theta_W}\right) - \lambda\right] \tag{1}$$

where y_t is the Higgs coupling to the top quark, g_W is the Higgs coupling to the W boson, θ_W is the Weinberg angle and λ is the Higgs self coupling. Λ_{SM} is the UV cutoff for the Standard Model. If the Standard Model is in fact an effective field theory then it is only valid up to a certain energy and distance (Λ_{SM}). Beyond this energy, we start to observe new physics.

The larger the cutoff, the greater the counter term required to cancel out the large correction and reproduce the mass of 125 GeV. This correction requires fine tuning to one part in 10^{24} [2]. This is the origin of the naturalness problem: such incredibly precise fine tuning seems unlikely to occur.

A Composite Higgs

A possible resolution is to treat the Higgs as a composite particle; specifically, a pseudo Nambu-Goldstone boson (pNGB). Goldstone's theorem states that when a group G of dimension $\dim(G)$ is spontaneously broken to H of dimension $\dim(H)$ there exist $\dim(G) - \dim(H)$ massless scalar particles known as Nambu-Goldstone bosons (NGBs). A pNGB corresponds to a softly broken symmetry that is spontaneously broken. An example is chiral symmetry in low energy Quantum Chromodynamics (QCD): the symmetry is inexact due to the (small) masses of the up and down quarks which causes the would-be NGBs to acquire a small mass; these are the pions ($\pi^{\pm}$ and π^0). Such a mechanism for the Higgs field is a more natural explanation for the Higgs boson's low mass compared to extreme fine-tuning.

The symplectic group $Sp(2N)$ is defined as follows:

$$Sp(2N) = \{M \in SU(2N) : M^* = \Omega^\dagger M \Omega\}$$

where $SU(2N)$ denotes the special unitary group of odd-rank and

$$\Omega = \begin{bmatrix} 0 & \mathbb{1}_N \\ -\mathbb{1}_N & 0 \end{bmatrix} \tag{2}$$

where $\mathbb{1}_N$ is the $N \times N$ identity matrix.

A model for a composite Higgs particle proposed in [3] is that of the $SU(4)/Sp(4)$ coset. There are $15 - 10 = 5$ pNGBs that result, four of which are interpreted as the familiar Higgs doublet. In addition, the pure Yang-Mills theory will give rise to a spectrum of massive particles composed solely of the gauge-bosons. These are known as glueballs in analogy with QCD.

Studying the glueball spectrum in the absence of fermionic matter and as a function of N allows us to characterise important dynamical aspects of $Sp(2N)$ gauge theories.

The Glueball Spectrum

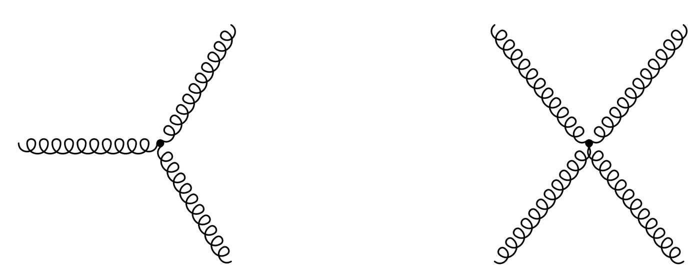

Figure 3: *Gluon self-interactions.*

The non-Abelian nature of Yang-Mills theory allows the gauge bosons to interact with each other as shown in figure 3. Their masses can be determined by lattice calculations. If the operator, $\Phi(0)$ creates a glueball state at time 0 and $\Phi^\dagger(t)$ annihilates same at time t then

$$\langle \Phi^\dagger(t)\Phi(0)\rangle = \sum_{n=1}^{\infty} |\langle n|\Phi(0)|0\rangle|^2 e^{-tE_n}$$

where $|n\rangle$ is an energy eigenstate with eigenvalue E_n. We define the vacuum as having zero energy: $E_0 = 0$.

At large values of t, the correlator $\langle \Phi^\dagger(t)\Phi(0)\rangle$ behaves like a single exponential and the exponent is related to the glueball's mass. Several sophisticated methods for improving the signal-to-noise ratio exist and are expatiated in [4, 5, 6]. The mass in the exponent is in units of lattice spacing. We recompute them in units of string tension σ.

It has also been demonstrated that the groups $SU(N)$, $Sp(2N)$ and $SO(N)$ all produce the same physics in the limit $N \to \infty$. Lattice results allow comparisons with field- and string-theories that also approach the same limit.

Results are shown in figure 4. It's clear from the plot that as $N \to \infty$, the glueball masses are the same for $SU(N)$ and $Sp(2N)$.

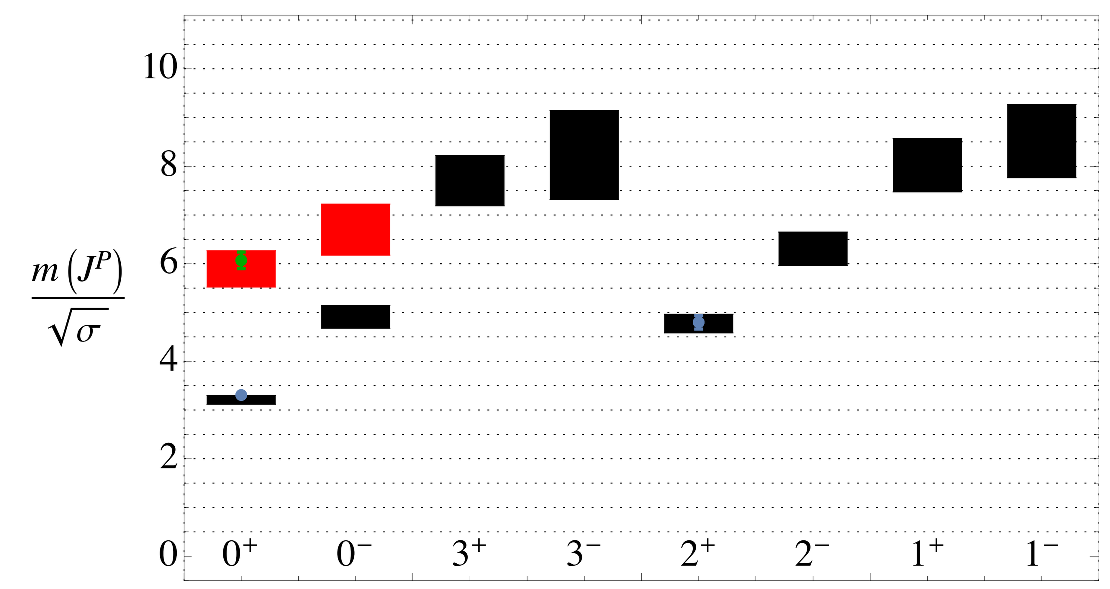

Figure 4: *Glueball spectrum as $N \to \infty$. Black boxes denote ground state $Sp(\infty)$ glueballs with first excitations coloured red. Blue dots denote ground state $SU(\infty)$ glueballs with first excitations coloured green. Each measurement's vertical width corresponds to the statistical error in the measurement.*

These results are very relevant for building effective models of Composite Higgs and of potential Dark Matter candidates.

References

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