CP violation in $K \rightarrow \pi\pi$ decays

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DiRAC Science Day,
Virtually, at the IPPP, Durham
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1. $K \rightarrow \pi\pi$ decays - introductory comments

- $|K^0\rangle = |\bar{s}d\rangle$, $|\pi^+\rangle = |\bar{d}u\rangle$ etc.

- $K \rightarrow \pi\pi$ decays are a very important class of processes for standard model phenomenology with a long and noble history.
  - It is in these decays that both indirect and direct CP-violation was discovered.

- Bose Symmetry $\Rightarrow$ the two-pion state has isospin 0 or 2.

  $I=2 \langle \pi\pi | H_W | K^0 \rangle = A_2 \ e^{i\delta_2} $, \quad $I=0 \langle \pi\pi | H_W | K^0 \rangle = A_0 \ e^{i\delta_0}$.

- The mathematics of isospin is the same as for the addition of angular momentum/spin in quantum mechanics. $(u, d)$ are an isospin 1/2 doublet and $s$ is a singlet. Pions have isospin 1 and the kaon has isospin 1/2.

- Among the very interesting issues are the origin of the $\Delta I = 1/2$ rule ($\text{Re} \ A_0/\text{Re} \ A_2 \approx 22.5$) and an understanding of the experimental value of $\varepsilon'/\varepsilon$, the parameter which was the first experimental evidence of direct CP-violation.
Outline of Talk

“Direct CP violation and the $\Delta I = 1/2$ rule in $K \rightarrow \pi\pi$ decay from the Standard Model,”

The release of this paper allows me to tell a coherent story of RBC-UKQCD’s long-standing project on $K \rightarrow \pi\pi$ decays.

Outline of talk

1. Introduction to CP violation and $K \rightarrow \pi\pi$ decay amplitudes
2. Evaluation of $A_2$
3. Evaluation of $A_0$
4. Conclusions and Outlook
The RBC & UKQCD collaborations

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- Chulwoo Jung
- Meifeng Lin
- Aaron Meyer
- Hiroshi Ohki
- Shigemi Ohta (KEK)
- Amarjit Soni

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- Tom Blum
- Dan Hoying (BNL)
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- Cheng Tu

**Edinburgh University**
- Peter Boyle
- Luigi Del Debbio
- Felix Erben
- Vera Gülpers
- Tadeusz Janowski
- Julia Kettle
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- Antonin Portelli
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- Andrew Yong
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- Julien Frison

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- Nicolas Garron

**MIT**
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**Peking University**
- Xu Feng

**University of Regensburg**
- Christoph Lehner (BNL)

**University of Southampton**
- Nils Asmussen
- Jonathan Flynn
- Ryan Hill
- Andreas Jüttner
- James Richings
- Chris Sachrajda

**Stony Brook University**
- Jun-Sik Yoo
- Sergey Syritsyn (RBRC)
Flavour physics - generalized $\beta$-decays

- At the level of quarks we understand nuclear $\beta$ decay in terms of the fundamental process:

\[
\begin{array}{c}
d \\
\rightarrow \\
u \\
\leftarrow \\
e^- \\
W \\
\leftarrow \\
\bar{\nu}
\end{array}
\]

- With the 3 generations of quarks and leptons in the standard model this is generalized to other *charged current* processes, e.g.:

\[
\begin{array}{c}
b \\
\rightarrow \\
c \\
\leftarrow \\
e^- \\
W \\
\leftarrow \\
\bar{\nu}
\end{array}
\]

\[
\begin{array}{c}
s \\
\rightarrow \\
u \\
\leftarrow \\
e^- \\
W \\
\leftarrow \\
\bar{\nu}
\end{array}
\]

\[
\begin{array}{c}
b \\
\rightarrow \\
u \\
\leftarrow \\
e^- \\
W \\
\leftarrow \\
\bar{\nu}
\end{array}
\]

- Weak interaction eigenstates \( \neq \) mass eigenstates:

\[
\begin{pmatrix}
d \\ s \\ b
\end{pmatrix} \rightarrow V_{\text{CKM}}
\begin{pmatrix}
u \\ c \\ t
\end{pmatrix}
\]

where \( V_{\text{CKM}} \) is a unitary \( 3 \times 3 \) matrix with 4 independent parameters, one of which is a (CP-violating) phase.
CP-violation in $K \rightarrow \pi\pi$ decays

\[ |K^0\rangle = |\bar{s}d\rangle \quad |\bar{K}^0\rangle = |s\bar{d}\rangle \]

$CP|K^0\rangle = -|\bar{K}^0\rangle$ (phase convention dependent)

- If CP were an exact symmetry of nature then the mass eigenstates would be $K_{1,2} = \frac{1}{\sqrt{2}} (|K^0\rangle \mp |\bar{K}^0\rangle)$.
- The $|\pi\pi\rangle$ state (in s-wave and isospin 0,2) is CP even.
- Similarly the $|\pi\pi\pi\rangle$ state is CP-odd.
- If CP were exact then $K_1 \rightarrow \pi\pi$ and $K_2 \rightarrow \pi\pi\pi$. This is almost, but not exactly, the case.
- What is observed experimentally is that

\[ \frac{\langle \pi^+\pi^- | H_W | K_L \rangle}{\langle \pi^+\pi^- | H_W | K_S \rangle} \simeq \frac{\langle \pi^0\pi^0 | H_W | K_L \rangle}{\langle \pi^0\pi^0 | H_W | K_S \rangle} \simeq \epsilon \]

where $|\epsilon| = (2.23 \pm 0.01) \times 10^{-3}$. J.Cronin & V.Fitch, 1964 (1980 Nobel Prize)

- Equating the above ratios assumes that CP is only broken in the eigenstates. Was it also broken in the decay?
- This was a question which took more than 30 years to answer experimentally.
CP-violation in $K \rightarrow \pi\pi$ decays (cont.)

- A more precise way of parametrising the ratios is:

$$
\eta_{+-} = \frac{\langle \pi^+\pi^- | H_W | K_L \rangle}{\langle \pi^+\pi^- | H_W | K_S \rangle} = \epsilon + \epsilon'
$$

$$
\eta_{00} = \frac{\langle \pi^0\pi^0 | H_W | K_L \rangle}{\langle \pi^0\pi^0 | H_W | K_S \rangle} = \epsilon - 2\epsilon'
$$

where $\epsilon'$ is a measure of CP-violation in the decays themselves.

Experimentally:

- Indirect CP-violation: $|\epsilon| = (2.228 \pm 0.011) \times 10^{-3}$
- Direct CP-violation: $\text{Re}(\epsilon'/\epsilon) = (16.6 \pm 2.3) \times 10^{-4}$. 

The aim of this project was to compute $\text{Re}(\epsilon'/\epsilon)$ from first principles and to provide a quantitative understanding of the $\Delta I = 1/2$ rule.

This has been by far the most complicated project in which I have been involved.
For illustration above are two contributions to $K^0 \rightarrow \pi^+\pi^-$ decays.

- Gluon exchanges are not shown.

The starting point is to organise the contributions into a form so that the decay amplitude can be computed; this is now well established.
Directly $CP$-violating decays are those in which a $CP$-even (-odd) state decays into a $CP$-odd (-even) one:

$$K_L \propto K_2 + \bar{\epsilon}K_1.$$ 

Consider the following contributions to $K \rightarrow \pi\pi$ decays:

\begin{itemize}
  \item (a) $I = 0$ or 2, Complex \hspace{1cm} \(\bar{s}\bar{d}d\bar{u}, \bar{c}\bar{t}d\bar{u}\)
  \item (b) $I = 0$, Real \hspace{1cm} \(\bar{s}\bar{d}d\bar{u}\)
  \item (c) $I = 0$ or 2, Real \hspace{1cm} \(\bar{s}\bar{u}d\bar{d}\)
\end{itemize}

\[
\text{Re} \left( \frac{\epsilon'}{\epsilon} \right) \propto \frac{\text{Im}A_2}{\text{Re}A_2} - \frac{\text{Im}A_0}{\text{Re}A_0}
\]
Lattice phenomenology starts with the evaluation of correlation functions of the form:

$$\langle 0 | O(x_1, x_2, \cdots, x_n) | 0 \rangle = \frac{1}{Z} \int [dA_\mu] [d\psi] [d\bar{\psi}] e^{-S} O(x_1, x_2, \cdots, x_n),$$

where $O(x_1, x_2, \cdots, x_n)$ is a multilocal operator composed of quark and gluon fields and $Z$ is the partition function.

The physics which can be studied depends on the choice of the multilocal operator $O$.

The functional integral is performed by discretising Euclidean space-time and using Monte-Carlo Integration.
Difficulties with two particles in the final state

- $K \rightarrow \pi\pi$ correlation function is dominated by the lightest state, i.e. the state with two-pions at rest (or the vacuum for $I = 0$). Maiani and Testa, PL B245 (1990) 585

\[ C(t_{\pi}) = A + B_1 e^{-2m_\pi(t_{\pi} - t_H)} + B_2 e^{-2E_\pi(t_{\pi} - t_H)} + \cdots \]

- Solution 1: Study an excited state. Lellouch and Lüscher, hep-lat/0003023
- Solution 2: Introduce suitable boundary conditions such that, after the vacuum subtraction, the $\pi\pi$ ground state is $|\pi(q)\pi(-q)\rangle$.
  

For $B$-decays, with so many intermediate states below threshold, this is the main obstacle to producing reliable calculations.
Imagine now that we chosen the boundary conditions so that the ground state is $|\pi(\vec{q})\pi(-\vec{q})\rangle$.

- In a finite volume each component of $\vec{q}$ is quantised, with allowed values separated by $2\pi/L$.
- Thus in order to obtain the physical value of $|\vec{q}|$ the volume must be chosen appropriately.
- Moreover, the s-wave, $I = 0$ and $I = 2$ channels are attractive and repulsive respectively and so the two cases must be studied on lattices of different volumes.
2. Results for $A_2$

- The amplitude $A_2$ is considerably simpler to evaluate than $A_0$.

- Our first results for $A_2$ at physical kinematics were obtained at a single, rather coarse, value of the lattice spacing ($a \simeq 0.14$ fm). Estimated discretization errors at 15%. (Winner of 2012 Ken Wilson Lattice Award.)

  - We found a surprising cancellation, and hence suppression, in Re$A_2$. This is an important element in understanding the $\Delta I = \frac{1}{2}$ rule.

- Our latest results were obtained on two new ensembles, $48^3$ with $a \simeq 0.11$ fm and $64^3$ with $a \simeq 0.084$ fm so that we can make a continuum extrapolation:

  \[
  \text{Re}(A_2) = 1.50(4)_{\text{stat}}(14)_{\text{syst}} \times 10^{-8} \text{ GeV},
  \]
  \[
  \text{Im}(A_2) = -6.99(20)_{\text{stat}}(84)_{\text{syst}} \times 10^{-13} \text{ GeV}.
  \]

  - The experimentally measured value is $\text{Re}(A_2) = 1.479(4) \times 10^{-8} \text{ GeV}$.

  - Although the precision can still be significantly improved (partly by perturbative calculations), the calculation of $A_2$ at physical kinematics can now be considered as standard.

  - We are not currently working towards improving this result.
3. Evaluation of $A_0$ and $\epsilon'/\epsilon$

- In 2015 RBC-UKQCD published our first result for $\epsilon'/\epsilon$ computed at physical quark masses and kinematics, albeit still with large relative errors:
  
  \[ \frac{\epsilon'}{\epsilon} \bigg|_{\text{RBC-UKQCD}} = (1.38 \pm 5.15 \pm 4.59) \times 10^{-4} \]

  to be compared with
  
  \[ \frac{\epsilon'}{\epsilon} \bigg|_{\text{Exp}} = (16.6 \pm 2.3) \times 10^{-4}. \]

- Is this 2.1σ deviation real? \(\Rightarrow\) must reduce the uncertainties.
- The matrix elements themselves are calculated with a smaller relative error.

- Puzzle: For the $I = 0$ s-wave $\pi\pi$ phase shift we obtained $\delta_0 = (23.8 \pm 4.9 \pm 2.2)^\circ$, to be compared with the dispersive results of about $34^\circ$.

  G.Colangelo et al.
From 2015 to 2020

2015

- $32^3 \times 64$ ensemble (Möbius DWF and Iwasaki + DSDR gauge action)
- $a^{-1} = 1.3784(68)$ GeV, $L = 4.53$ fm.
- G-parity boundary conditions in 3-directions
- 216 configurations
- Almost physical kinematics: $(m_\pi = 143.1(2.0)$ MeV, $m_K = 490.6(2.2)$ MeV, $E_{\pi\pi} = 498(11)$ MeV).

Extension and Improvement in 2020

- Increase the statistics: 216 $\rightarrow$ 1438 configurations.
  - Reduce the statistical error;
  - Improved statistics allows for an in-depth study of the systematics.
- Use an expanded set of operators to create the $\pi\pi$ state. (741 configurations)
- Improve the non-perturbative renormalisation, including step-scaling to match at a higher energy.
- Significantly improve the analysis techniques.

C.Kelly and T.Wang, arXiv:1911.04582
Increasing the statistics from 216 to 1438 configurations, the $\pi\pi$ correlation function is still well described by a single $\pi\pi$ state.

- It does not solve the $\delta_0$ puzzle however:

$$\delta_0 = (23.8 \pm 4.9 \pm 2.2)^\circ \rightarrow \delta_0 = (19.1 \pm 2.5 \pm 1.2)^\circ \quad (\chi^2/\text{dof} = 1.6)$$
The $\delta_0$-puzzle has been resolved by adding more interpolating operators for the $\pi\pi$ states. Originally we only had a single $\pi\pi$ operator with each pion being given a momentum $\pm (1, 1, 1)\pi/L$ (with total momentum $\vec{0}$).

In particular the inclusion of a $\sigma$-like two-quark operator $(\bar{u}u + \bar{d}d)$ has exposed a second state, e.g. for $t_f - t_i = 5$

$$\det \begin{pmatrix} \langle \pi\pi(t_f)\pi\pi(t_i) \rangle & \langle \pi\pi(t_f)\sigma(t_i) \rangle \\ \langle \sigma(t_f)\pi\pi(t_i) \rangle & \langle \sigma(t_f)\sigma(t_i) \rangle \end{pmatrix} = 0.439(50) \neq 0$$

We have also included a third operator giving each pion a larger momentum $\pm (3, 1, 1)\pi/L$.

At present we have only analysed 741 configurations with the additional operators. Remainder will be done in the future.


\[ \delta_0 = (32.3 \pm 1.0 \pm 1.8)^\circ \text{ from a fit in the range } t = 5 - 15 \text{ (statistical error only).} \]

The fit from dispersion theory at this value of \( E_{\pi\pi} \) and \( m_\pi \) is about 35.9°.

The \( \pi\pi(3, 1, 1) \) operator turns out not to be very important.
Changes in estimates of systematic errors since 2015 calculation

<table>
<thead>
<tr>
<th>Description</th>
<th>2015 Error</th>
<th>2020 Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operator normalisation</td>
<td>15%</td>
<td>5%</td>
</tr>
<tr>
<td>Wilson coefficients</td>
<td>12%</td>
<td>unchanged</td>
</tr>
<tr>
<td>Finite lattice spacing</td>
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<td>unchanged</td>
</tr>
<tr>
<td>Lellouch - Lüscher factor</td>
<td>11%</td>
<td>1.5%</td>
</tr>
<tr>
<td>Residual FV corrections</td>
<td>7%</td>
<td>unchanged</td>
</tr>
<tr>
<td>Parametric errors</td>
<td>5%</td>
<td>6%</td>
</tr>
<tr>
<td>Excited state contamination</td>
<td>5%</td>
<td>negligible</td>
</tr>
<tr>
<td>Unphysical kinematics</td>
<td>3%</td>
<td>5%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>27%</strong></td>
<td><strong>21%</strong></td>
</tr>
</tbody>
</table>

1. As a result of step scaling from $\mu = 1.53 \text{ GeV} \rightarrow 4.00 \text{ GeV}$.
2. Better control of $\pi\pi$ system due to additional operators.
3. Largest uncertainty is due to $\tau \sim 5\%$.
Principal Results

- \( \text{Re} A_0 = 2.99(0.32)(0.59) \times 10^{-7} \text{GeV} \) (Experiment: 3.3201(18) GeV)

- \( \text{Im} A_0 = -6.98(0.62)(1.44) \times 10^{-11} \text{GeV} \).

Combining our new result for \( \text{Re} A_0 \) with our 2015 one for \( \text{Re} A_2 \) we find:

\[
\frac{\text{Re} A_0}{\text{Re} A_2} = 19.9 \pm 2.3 \pm 4.4,
\]

in good agreement with the experimental result of 22.45(6).

Combining our new result for \( \text{Im} A_0 \) with our 2015 result for \( \text{Im} A_2 \) and using the experimental results for the real parts, we find

\[
\text{Re} \left( \frac{\epsilon'}{\epsilon} \right) = 0.00217(26)(62)(50),
\]

where the third uncertainty is due to isospin breaking effects. This result is consistent with the experimental value of 0.00166(23).

- Note that if, instead of treating the isospin correction from arXiv:1911.01359 as a component of the systematic uncertainty, we were to implement on our result, we would obtain a central value \( \text{Re}(\epsilon'/\epsilon) = 0.00167 \), coincidentally identical to the experimental result.
4 Conclusions and Outlook

- We have completed the update on our 2015 lattice determination of $A_0$ and $\epsilon'/\epsilon$ with:
  - a 3.2 times increase in statistics;
  - the use of multi-operator techniques in order to essentially remove the systematic error due to excited state contamination;
  - the use of step-scaling to reduce significantly the systematic error in the renormalisation.

- We reproduce the experimental value of $\text{Re}A_0/\text{Re}A_2$ demonstrating that, within our uncertainties, QCD is sufficient to solve the decades-old puzzle of the $\Delta I = 1/2$ rule.
  - We had already previously found a surprising cancellation, and hence suppression, in $\text{Re}A_2$. P.A.Boyle et al., arXiv:1212.1474

- Our result for $\text{Re} \epsilon'/\epsilon$ is consistent with the experimental value, with an error which is about 3.5 times larger. This quantity remains a promising avenue in which to search for new physics but more precision is required.
The collaboration intends to perform measurements on two larger lattices with different lattice spacings to perform the continuum limit. This will require the next generation of supercomputers.

A project is currently underway to perform the $4 \rightarrow 3$ flavour matching in the Wilson coefficients non-perturbatively. M. Tomii, arXiv:1901.04107

We are also working on laying the groundwork for the lattice calculation of isospin breaking and electromagnetic effects. N. Christ and Xu Feng, arXiv:1711.09339

The collaboration is actively investigating the potential for multi-operator fits to circumvent need for G-parity BCs, allowing for more reuse of ensembles and eigenvectors from other RBC&UKQCD projects. D. Hoying, PoS LATTICE2018 (2019) 064